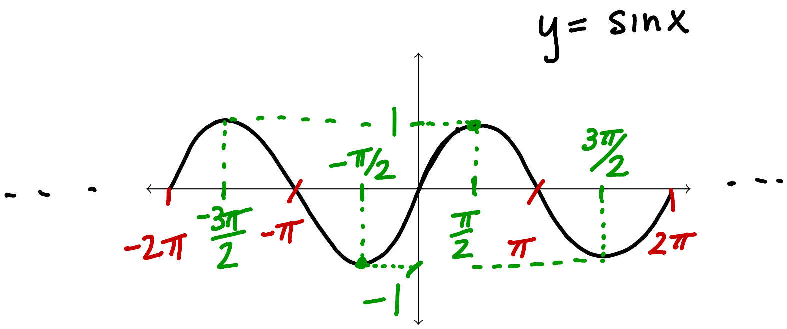
Review Day 3: Trigonometry Review

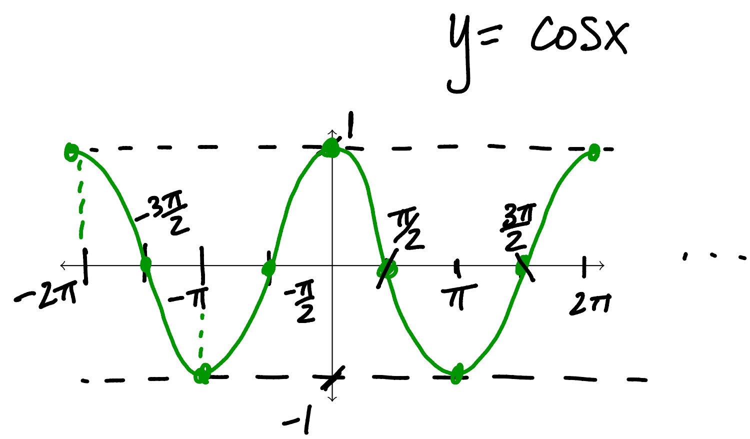
Three Views of Trigonometric Functions

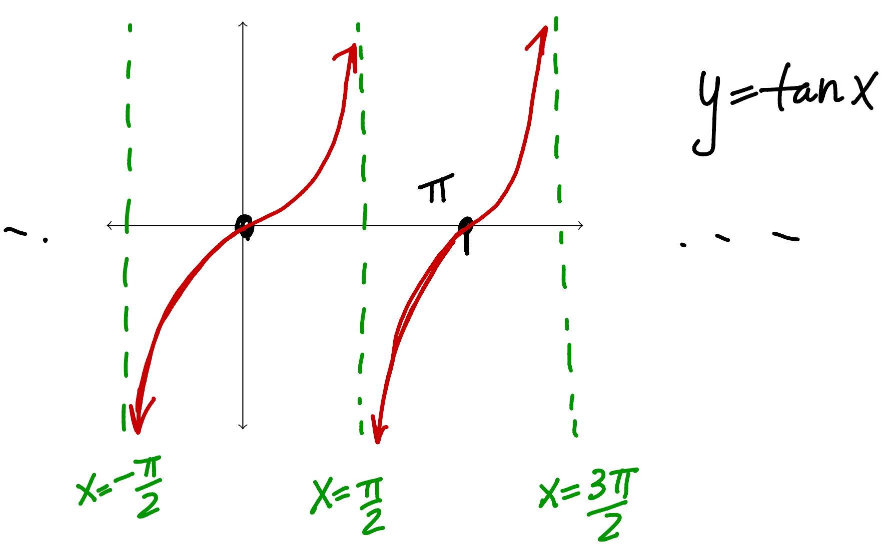
- graphs in the xy-plane
- sides of a right triangle
- points on the unit circle

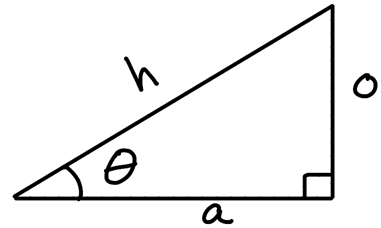
The Graphs

On the axes below, graph *at least two cycles* of $f(x) = \sin x$, $f(x) = \cos x$, and $f(x) = \tan x$. **Label all x- and y-intercepts, any asymptotes, and all maximums and minimums.**









The Triangle Defintion

Sketch a right triangle with side a adjacent to an angle θ, o opposite of the angle θ and hypotenuse h. Define each of the six trigonometric functions in terms of that triangle.

a) $\sin\theta$ $= \dfrac{o}{h}$

b) $\cos\theta$ $= \dfrac{a}{h}$

c) $\tan\theta$ $= \dfrac{o}{a}$

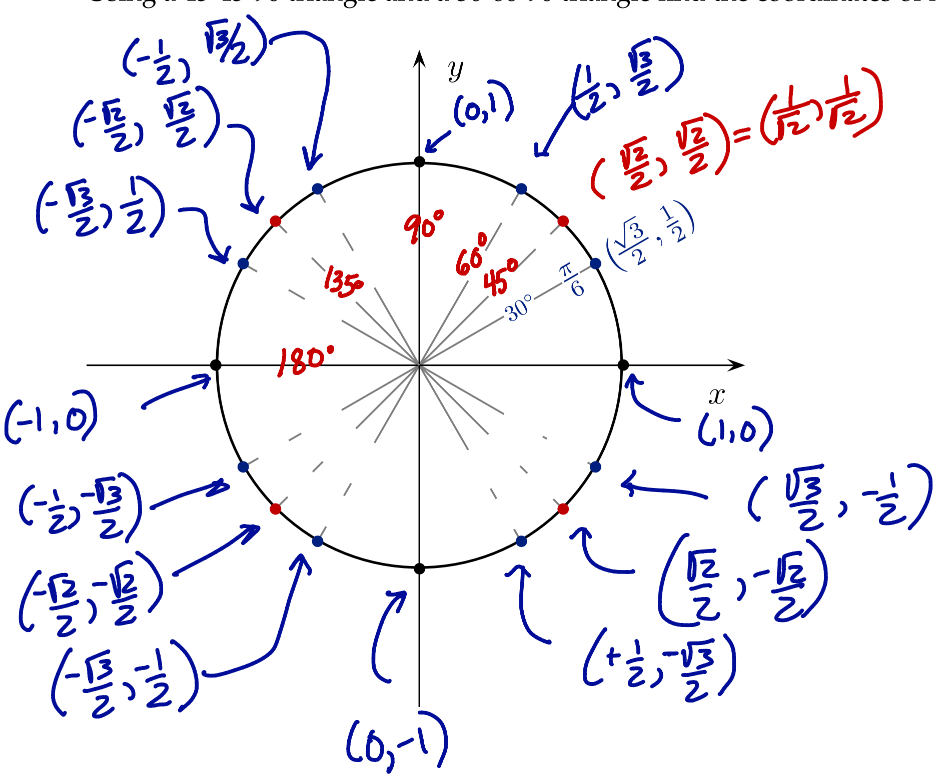
d) $\sec\theta$ $= \dfrac{1}{\cos\theta} = \dfrac{h}{a}$

e) $\csc\theta$ $= \dfrac{1}{\sin\theta} = \dfrac{h}{o}$

f) $\cot\theta$ $= \dfrac{1}{\tan\theta} = \dfrac{a}{o}$

The Unit Circle Approach

Using a 45-45-90 triangle and a 30-60-90 triangle find the coordinates of ALL of the points on the unit circle.



ALSO Know:

$30° = \frac{\pi}{6}$ rad

$45° = \frac{\pi}{4}$ rad

$60° = \frac{\pi}{3}$ rad

$90° = \frac{\pi}{2}$ rad

⋮

Conversion?

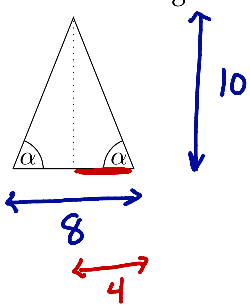
$360° = 2\pi$ rad

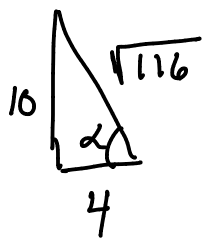
So...

$\dfrac{180°}{\pi \text{ rad}}$ or $\dfrac{\pi \text{ rad}}{180°}$

Each of the problems below can be solved using one of the approaches above: graphs, triangles, or unit circle. When you solve each problem, think about which method is the best one.

1. An isosceles triangle has a height of 10 ft and its base is 8 feet long. Determine the sine, cosine and tangent of the base angle α.





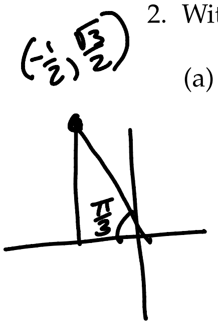
$\sin(\alpha) = \frac{o}{h} = \frac{10}{\sqrt{116}}$

$\cos(\alpha) = \frac{a}{h} = \frac{4}{\sqrt{116}}$

$\tan\theta = \frac{4}{10} = \frac{2}{5}$

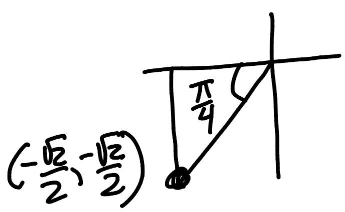
2. Without a calculator evaluate:

(a) $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$

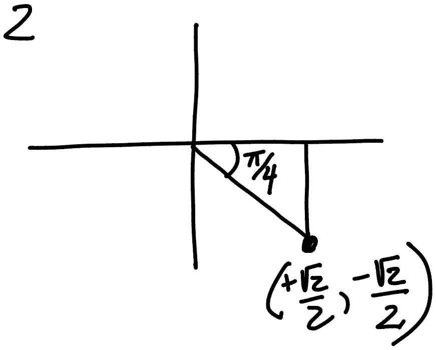


(b) $\cos(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\frac{5\pi}{4} = \pi + \frac{\pi}{4}$

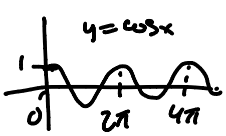


(c) $\tan(\frac{-\pi}{4}) = -1$



3. Solve for x.

(a) $\cos x = 1$



$x = 2\pi k$, k integer

OR

$x = \ldots -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$

(b) $\sin x = 1$

$x = 2\pi k + \frac{\pi}{2}$, k integer

OR

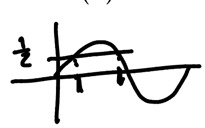
$x = \ldots -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

(c) $\tan x = 0$

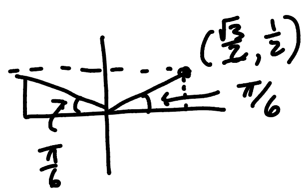
$x = \ldots -\pi, 0, \pi, 2\pi, \ldots$

(from graph!)

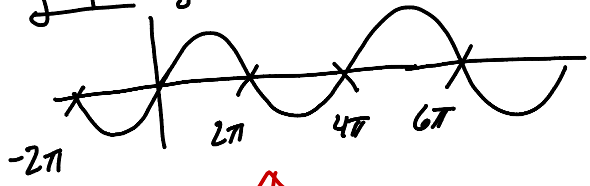
(d) $\sin x = 1/2$ (Find all solutions in $[0, 2\pi]$.)



$x = \frac{\pi}{6}, \frac{5\pi}{6}$



graph $y = \sin(x/2)$



graphically

4. Find the domain of $f(x) = \csc(x/2)$. $= \dfrac{1}{\sin(\frac{x}{2})}$

We need to find where $\sin\left(\frac{x}{2}\right) = 0$

algebraically

We know $\sin(\theta) = 0$ when $\theta = \pi \cdot k$, k integer.

So we find x where $\frac{x}{2} = \pi k$ or $x = 2\pi k$

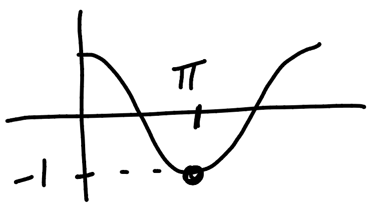
Answer: Domain $f(x)$ is all real numbers except $x = 2\pi k$, k integer

$\ldots (-2\pi, 0) \cup (0, 2\pi) \cup (2\pi, 4\pi) \cup \ldots$

5. Solve the equation $2 + 2\cos(x) = 0$.

$2\cos x = -2$

$\cos x = -1$



$x = \pi$ or $2\pi + \pi$ or $4\pi + \pi \ldots$

ans: $x = 2\pi k + \pi$ for k integer

OR

$x = \ldots -\pi, \pi, 3\pi, 5\pi, \ldots$

6. Find the domain of $g(x) = \sqrt{\sin(x-1) - 1}$.

We need $\sin(x-1) - 1 \geq 0$ or $\sin(x-1) \geq 1$

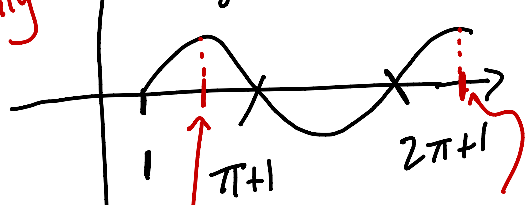
algebraic

(But $\sin(\theta) \leq 1$ (!!)) So we need $\sin(x-1) = 1$. We know $\sin\theta = 1$ when $\theta = \pi/2 + 2\pi k$.

So we need $x - 1 = \frac{\pi}{2} + 2\pi k$ or $x = \frac{\pi}{2} + 1 + 2\pi k$, k integer

graphically

$y = \sin(x-1)$



$\frac{\pi}{2} + 1$ $\frac{\pi}{2} + 1 + 2\pi$