1. State the definition of the derivative of a function $f(x)$ at $x = a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

2. Let $f(x) = 2x - \frac{2}{x}$.

(a) Use the definition to find the derivative of $f'(a)$.

$$f'(a) = \lim_{h \to 0} \frac{\left[2(a+h) - \frac{2}{a+h}\right] - \left[2a - \frac{2}{a}\right]}{h}$$

$$= \lim_{h \to 0} \frac{2a + 2h - 2a + \frac{2}{a} - \frac{2}{a+h}}{h} = \lim_{h \to 0} \left[2 + \frac{1}{h}\left(\frac{2a + 2h - 2a}{(a)(a+h)}\right)\right]$$

$$= \lim_{h \to 0} \left[2 + \frac{2}{(a)(a+h)}\right] = 2 + \frac{2}{a^2}$$

(b) (Your answer to part (a) should have been $f'(a) = 2 + \frac{2}{a^2}$. Find the slope of the tangent line to $f(x)$ when $x = -1$.

$$f'(-1) = 2 + \frac{2}{(-1)^2} = 4$$

(c) Write the equation of the line tangent to $f(x)$ when $x = -1$.

$$f(-1) = 2(-1) - \frac{2}{(-1)} = -2 + 2 = 0$$

$$y - 0 = 4(x+1) \quad \text{or} \quad \boxed{y = 4x + 4}$$

3. Suppose N represents the number of people in the United States who travel by car to another state for a vacation this Memorial Day weekend when the average price of gasoline is p dollars per gallon.

   (a) What are the units of dN/dp?

   people/\$per gal.

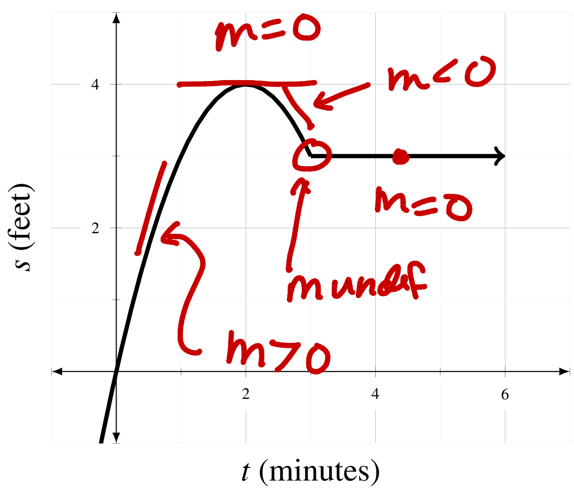
   (b) In the context of the problem, write a sentence interpreting $\frac{dN}{dp}$.

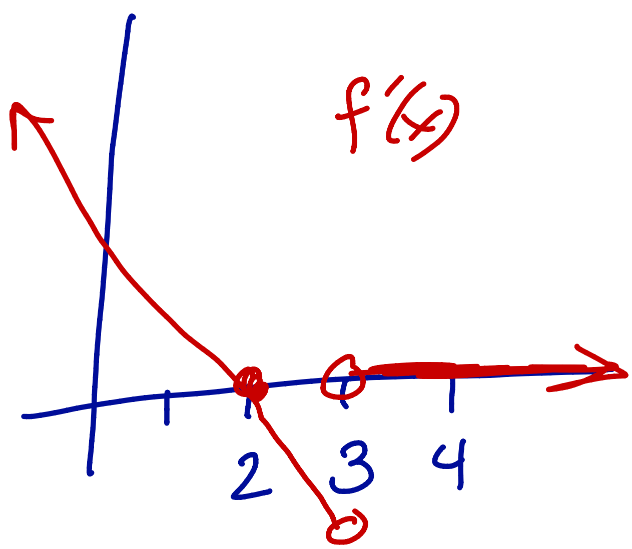
   $\frac{dN}{dp}$ gives the instantaneous rate of change of people relative to a change in price of gas.

   (c) Would you expect dN/dp to be positive or negative? Explain your answer.

   Negative. As the price of gas goes up, I would expect the # of people driving to decrease.

4. The graph of $f(x)$ is sketched below. On a separate set of axes, give a rough sketch $f'(x)$.

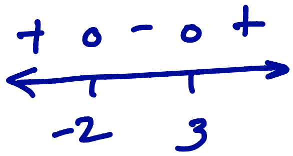




5. Find the domain of each function. Write your answer in interval notation.

   (a) $f(x) = \sqrt{x^2 - x - 6}$

   need $x^2 - x - 6 > 0$
   or $(x-3)(x+2) > 0$



   ANS:
   $(-\infty, -2) \cup (3, \infty)$

   (b) $g(t) = \ln(t + 6)$

   $t + 6 > 0$ so $t > -6$
   Ans: $(-6, \infty)$

6. State the definition of "The function $f(x)$ is continuous at $x = a$".

$$\lim_{x \to a} f(x) = f(a)$$

7. Suppose

$$f(x) = \begin{cases} -\frac{2}{x} & x < 2 \\ \frac{x}{x-3} & x \geq 2 \end{cases}$$

Is $f(x)$ continuous at $x = 0$? At $x = 2$? Justify your answers using the definition of continuity.

at $x=0$? No. f is undefined at $x=0$

at $x=2$? No.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{-2}{x} = -1 \neq -2 = \lim_{x \to 2^+} \frac{x}{x-3} = \lim_{x \to 2^+} f(x)$$

8. Find the limit or show that it does not exist. *Make sure you are writing your mathematics correctly and clearly.*

(a) $\displaystyle\lim_{x \to \infty} \frac{\sqrt[3]{8x^3+1}}{2-5x}$ $\cdot \frac{1/x}{1/x} = \displaystyle\lim_{x \to \infty} \frac{\sqrt[3]{8 + 1/x^3}}{\frac{2}{x} - 5} = \frac{2}{-5} = \frac{-2}{5}$

$\frac{4}{0}$ (b) $\displaystyle\lim_{r \to 16^-} \frac{\sqrt{r}}{(r-16)^3} = -\infty$

as $r \to 16^-$ $(r \approx 15.9)$, $\sqrt{r} \to 4 > 0$

$r - 16 \to 0^- < 0$

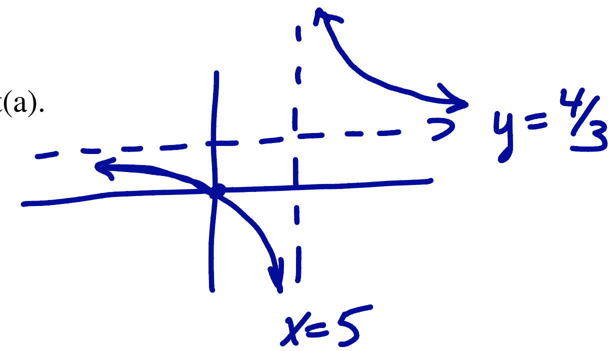
(c) $\displaystyle\lim_{x \to -3} \frac{x^2-9}{x^2+2x-3} = \lim_{x \to -3} \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-1)} = \lim_{x \to -3} \frac{x-3}{x-1} = \frac{-6}{-4}$

$= \frac{3}{2}$

9. (a) Write a formula for a function with a horizontal asymptote at $y = 4/3$ and a vertical asymptote at $x = 5$.

$$f(x) = \frac{4x}{3(x-5)}$$

(b) Sketch the graph of your function from part(a).



(c) Use limits to demonstrate that your function really does have a vertical asymptote at $x = 5$

$$\lim_{x\to 5^+} \frac{4x}{3(x-5)} = +\infty$$

b/c as $x \to 5^+$ ($x \approx 5.1$)

$4x \to 20 > 0$

$3(x-5) \to 0^+ > 0$

(d) Use limits to demonstrate that your function really does have a horizontal asymptote at $y = 4/3$.

$$\lim_{x\to\infty} \frac{4x}{3(x-5)} = \lim_{x\to\infty} \frac{4}{3-\frac{15}{x}} = \frac{4}{3-0} = \frac{4}{3}$$

10. Use the Intermediate Value Theorem to show $\ln x = x - 5$ has a solution. (Hint: Show there is a solution in the interval $[1, e^5]$.)

Let $f(x) = \ln x - x + 5$. Note $f(x)$ is continuous.

Now, $f(1) = \ln 1 - 1 + 5 = 4 > 0$ and $f(e^5) = 5 - e^5 + 5$
$> 22 > 0$.

Since f is positive at $x=1$ and negative at $x=e^5$, $f(x)$ must equal zero somewhere in between.