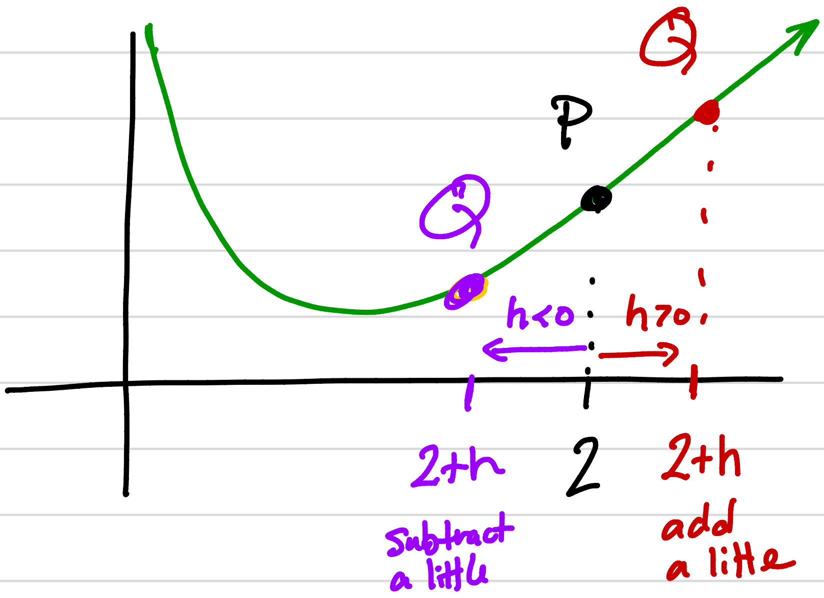
§ 2.7 Derivatives and Rates of Change

Recall

$f(x) = x + \frac{2}{x}$

Via a chart, we estimated the slope of the graph of $f(x)$ at $x=2$ is: $m = \frac{1}{2}$

We have better tools now!



$P(2,3)$ $Q(2+h, f(2+h))$

In either case,

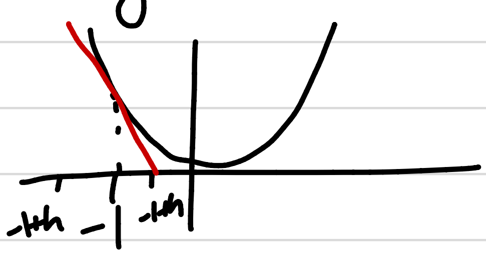
$$m_{sec} = \underset{\overline{PQ}}{\text{slope of secant}} = \frac{f(2+h) - 3}{(2+h) - (2)}$$

$$= \frac{2+h+\frac{2}{2+h} - 3}{h} = \frac{\frac{(h-1)(h+2)+2}{h+2}}{h}$$

$$= \frac{h^2+h-2+2}{h(h+2)} = \frac{h(h+1)}{h(h+2)} = \frac{(h+1)}{h+2}$$

$$m_{tan} = \lim_{h\to 0} m_{sec} = \lim_{h \to 0} \frac{h+1}{h+2} = \frac{1}{2}$$

Ex| Use this method to find slope of tangent (m_{tan}) to the graph of $f(x) = 3x^2$ at $x=-1$.



$P(-1,3)$ $Q(-1+h, 3(-1+h)^2)$

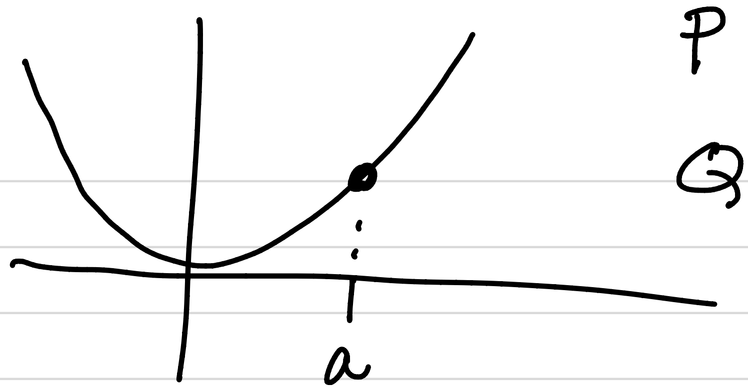
$$m_{sec} = \frac{3(-1+h)^2 - 3}{1+h-1} = \frac{3-6h+3h^2-3}{h} = \frac{3h(-2+h)}{h}$$

$$= 3(h-2)$$

$$m_{tan} = \lim_{h\to 0} m_{sec} = \lim_{h\to 0} 3(h-2) = -6$$

plausible?

What if we replace $x=-1$, with $x=a$?



$P(a, 3a^2)$

$Q(a+h, 3(a+h)^2)$

$$m_{sec} = \frac{3(a+h)^2 - 3a^2}{a+h-a} = \frac{3a^2 + 6ah + 3h^2 - 3a^2}{h} = \frac{3h(2a+h)}{h}$$

$$= 3(2a+h)$$

$$m_{tan} = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} 3(2a+h) = 6a \leftarrow \text{Wowee!}$$