Notes for Mon 17 Feb

topics § 3.2 Product + Quotient Rules

§ 3.3 Derivatives of Trig Functions

Goals of today: Proof, Plausibility, + Judgement

$\boxed{A}$ State formally the Product + Quotient Rules

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f'\cdot g + g'\cdot f$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g\cdot f' - f\cdot g'}{(g)^2}$$

$\boxed{B}$ Look, then compute derivatives for each function below

(i) $y = \frac{e^x+1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1}(e^x+1)$; $y' = \left(\frac{1}{\sqrt{2}+1}\right)e^x = \frac{e^x}{\sqrt{2}+1}$

(ii) $f(x) = \frac{x}{x+1}$; $f'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

(iii) $H(s) = \frac{A}{B\sqrt{s}} = \left(\frac{A}{B}\right)s^{-1/2}$; $H'(s) = \frac{-A}{2B}s^{-3/2}$

(iv) $y = (x-\pi)e^x$ $\quad y' = 1\cdot e^x + (x-\pi)e^x$

$= e^x(1+x-\pi)$

* Lesson Learned: Because you have a product or quotient does NOT mean you need to use the Product or Quotient Rules (!!!)

* You can exploit this to check your formulas.

Example: $y = \frac{10}{x^2} = 10x^{-2}$; $\underline{\underline{y' = -20x^{-3}}}$ ← use it to check your quotient rule.

We're certain of this.

Why Product Rule works the way it does?

Proof: $H(x) = f(x)g(x)$

$$H'(x) = \lim_{h\to 0} \frac{H(x+h) - H(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x) + \overbrace{f(x)g(x+h) - f(x)g(x+h)}^{\text{total of ZERO}}}{h}$$

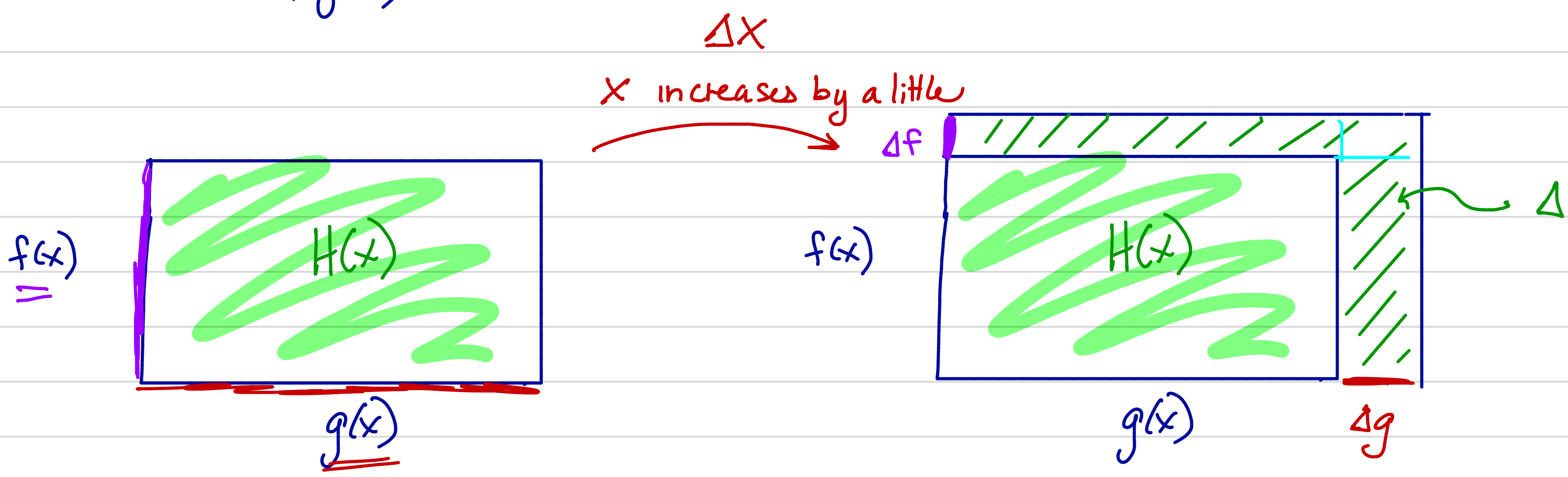
$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h)}{h} + \frac{f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \left[g(x+h)\left[\frac{f(x+h)-f(x)}{h}\right] + f(x)\left[\frac{g(x+h)-g(x)}{h}\right] \right]$$

$$= g(x)\cdot f'(x) + f(x)\cdot g'(x)$$

Intuitive Argument for Product Rule (Simultaneously a USEFUL way to think about Calculus ideas.)

$H(x) = f(x)\,g(x)$ area of rectangle...



So $\Delta H = g \cdot \Delta f + f \cdot \Delta g + \Delta f \cdot \Delta g$

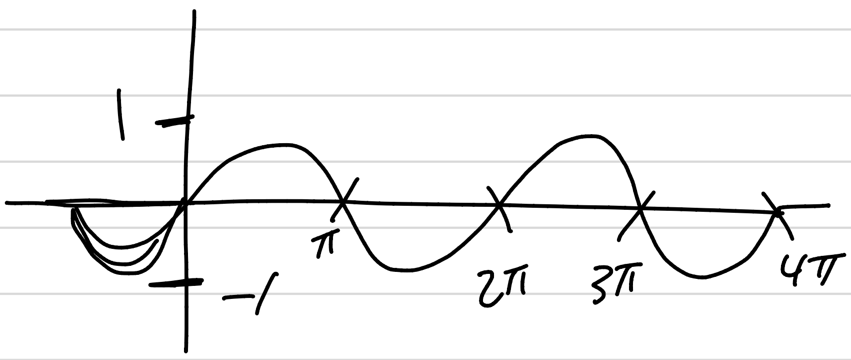
$$\frac{\Delta H}{\Delta x} = g \cdot \frac{\Delta f}{\Delta x} + f \cdot \frac{\Delta g}{\Delta x} + \frac{\Delta f \cdot \Delta g}{\Delta x}$$

$$H'(x) = g \cdot f' + f \cdot g' + 0$$

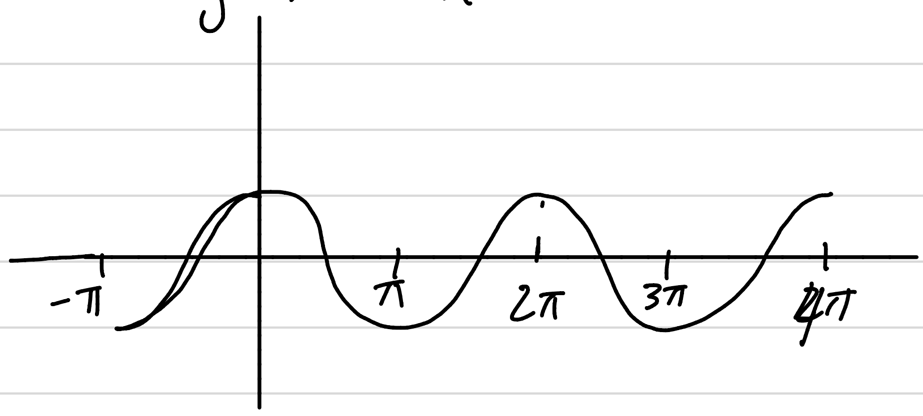
§ 3.3 Derivatives of Trig Functions

Sketch: on $[-\pi, 4\pi]$

$f(x) = \sin x$

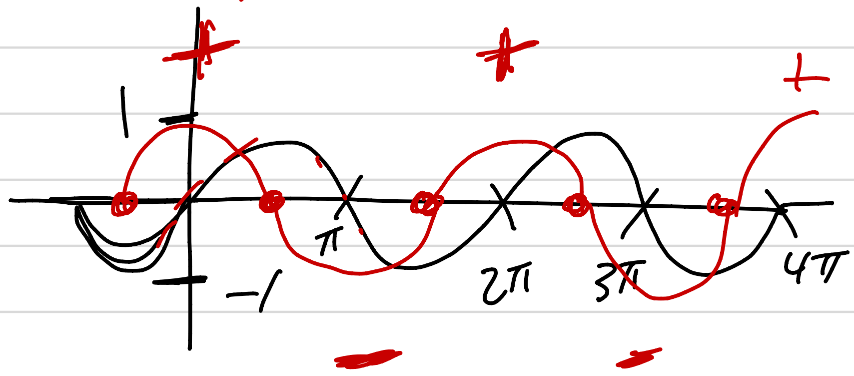


$g(x) = \cos x$



$f(x) = \sin x$

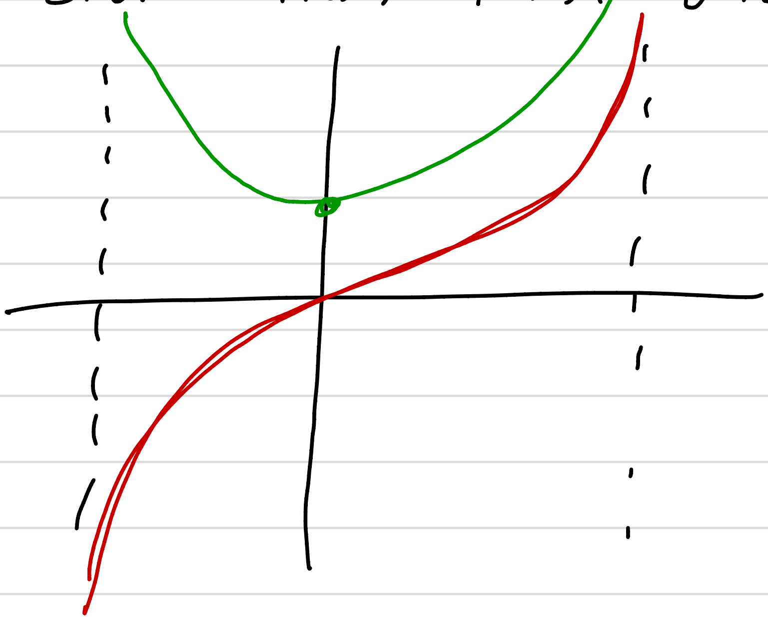
$f'(x) \leftarrow$ looks cosine-ish



$g(x) = \cos x$

$g'(x) \leftarrow$ looks ~~-~~ sine-ish

Sketch $h(x) = \tan x$ on $[-\pi/2, \pi/2]$



$h'(x)$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = \sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{(\cos x)(\sin x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$f(x) = \sin x$

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

trig id:
$\sin(a+b) = \sin a \cos b + \sin b \cos a$

$$\lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h \to 0} \sin x \left(\frac{\cos(h) - 1}{h}\right) + \cos x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \sin x \cdot 0 + \cos x \cdot 1$$