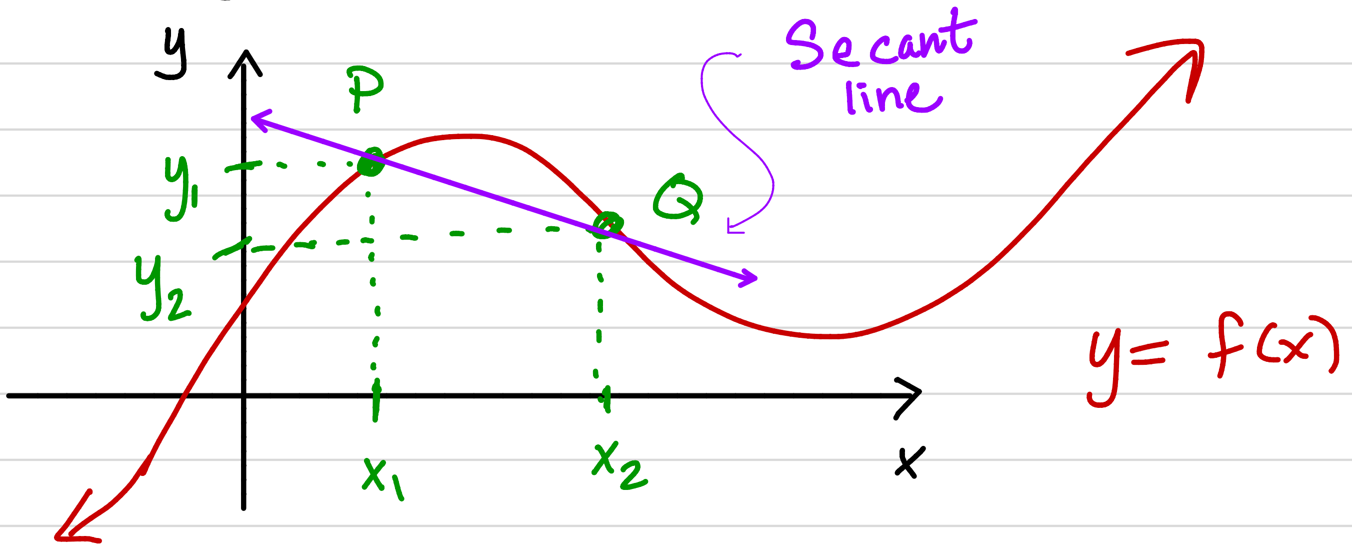
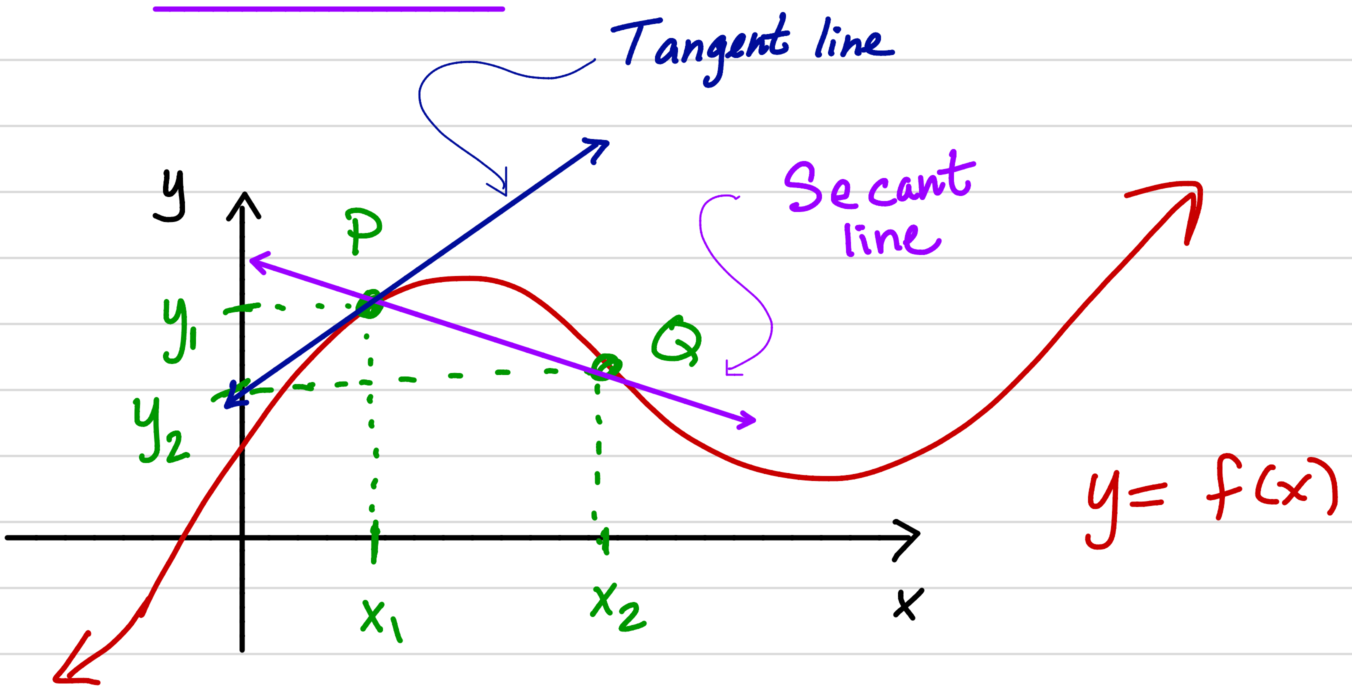
Section 2.1



Secant line means the line between two points on a graph.



tangent line is a line through one point of graph that "matches" the slope of the graph at that point

Crucial Ideas

1. Finding the slope of a line through two points is Easy. Through one point? Not easy
2. The tangent line can be approximated really well by a secant line.

Example: $f(x) = x^2$

1. Find slope of secant line from $x=1$ to $x=2$.

   ans: $f(1) = 1^2 = 1$, $f(2) = 2^2 = 4$; $P = (1,1)$, $Q = (2,4)$

   $m = \frac{4-1}{2-1} = \frac{3}{1} = \boxed{3}$

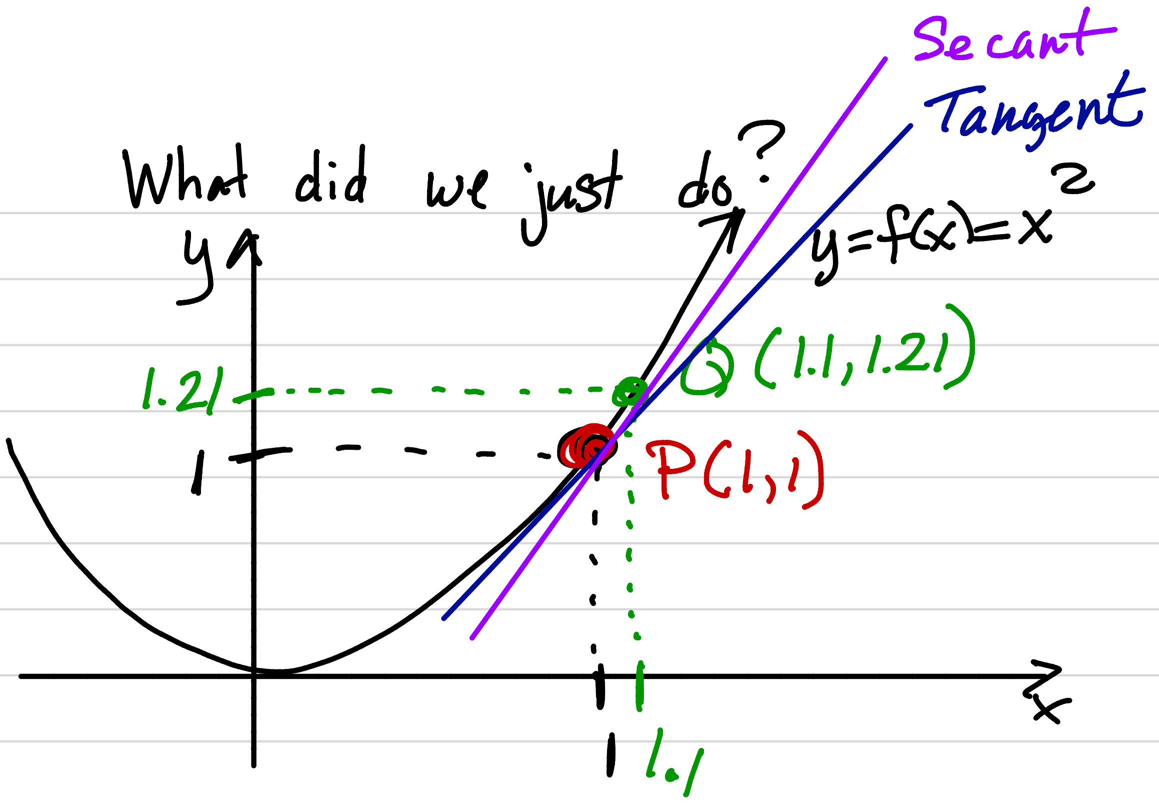
See cartoon

2. Use a secant line to estimate the slope of tangent line to $f(x)$ at $x=1$.

   ans: Since $P(1,1)$, pick a Q super-duper close ... like $x = 1.1$. So $y = (1.1)^2 = 1.21$. OR $Q(1.1, 1.21)$

   Now $m = \frac{1.21-1}{1.1-1} = \frac{0.21}{0.1} = 2.1$. So $m_{tangent} \boxed{\approx 2.1}$

What did we just do?



] ← pretty darn close

$m_{sec} \approx m_{tan}$

↑ roughly the same.

- How could we make our estimation better?
- Could someone else correctly answer the question slightly differently?

- Why would one care?

  What if $y=f(x)$ was distance travelled (in ft?) and x was time in (in sec?), what is m?

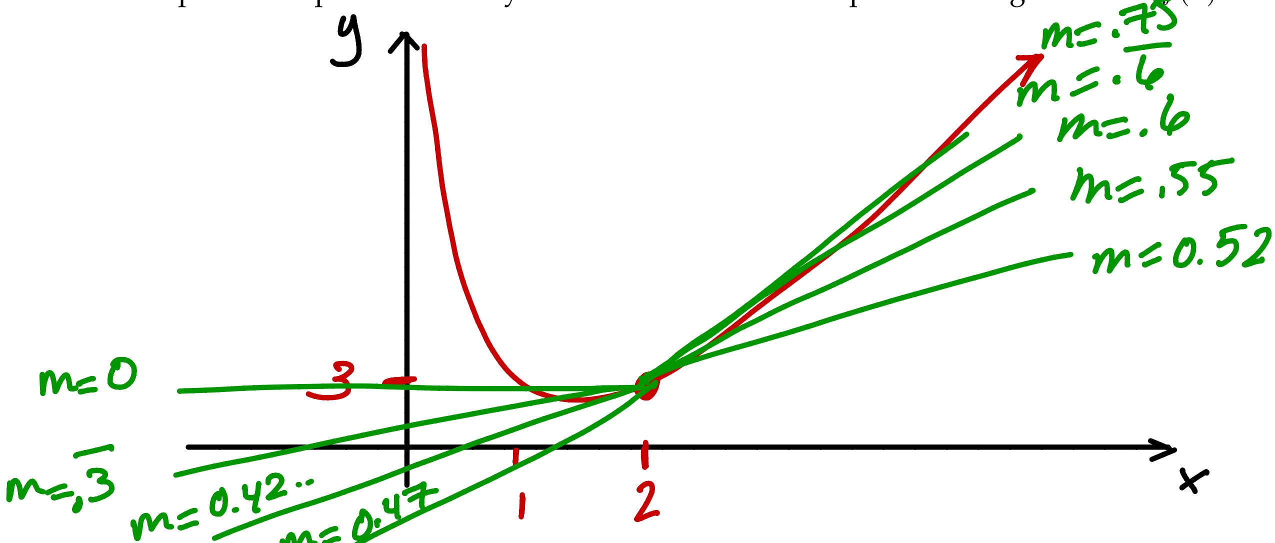
  What if $y=$ # heartbeats
  $x=$ time in seconds, what is m?

1. The point $P(2,3)$ lies on the graph of $f(x) = x + \frac{2}{x}$.

   (a) If possible, find the slope of the secant line between the point P and each of the points with x values listed below. For each estimate the slope to 4 decimal places. NOTE: You do not need the graph of the function to answer this numerical question.

point Q		slope of secant line PQ
x-value	y-value	PQ
$x = 4$	4.5	0.7500
$x = 3$	$3.\overline{6}$	$0.\overline{6}$
$x = 2.5$	3.3	0.6000
$x = 2.25$	3.1388	0.5555...
$x = 2.1$	3.05238	0.52380
$x = 0$	undefined	~
$x = 1$	3	0
$x = 1.5$	$2.8\overline{3}$	$0.\overline{3}$
$x = 1.75$	2.892857	0.42857
$x = 1.9$	2.95263	0.47368

   (b) Now, use technology to sketch a rough graph $f(x)$ on the interval $(0,5]$ and add the secant lines from part a. (Your graph may be messy...It's ok.) Label the secant lines with their respective slopes. What can you conclude about the slope of the tangent line to $f(x)$ at $x = 2$?



   (c) Write a best guess for the equation of the line tangent to $f(x)$ at point P. Is your equation plausible?

guess $m = \frac{1}{2}$. line: $y - 3 = \frac{1}{2}(x-2)$

$\boxed{y = \frac{1}{2}x + 2}$

Plausible? Yes. It should be positive (sloped up) and less than one.

2. The table shows the position of a cyclist after accelerating from rest.

t (minutes)	0	30	60	90	120	150	180	210	240
d (miles)	0	9.2	18.7	23.1	38.1	46.6	59.7	72.6	80

9.2   9.5   4.4   15   8.5   13.1   12.9   7.4

(a) Estimate the cyclist's average velocity in miles per hour during:

i. the first hour

$P(0,0)$, $Q(60,18.7)$ $m = \text{avg vel} = \dfrac{18.7}{60} = 18.7\ mi/hr.$

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ii. the second hour

$P(60,18.7)$ $Q(120,38.1)$ $m = \text{avg vel} = \dfrac{38.1-18.7}{60} = 19.4\ mi/hr$

iii. the third hour

$P(120,38.1)$
$Q(180,59.7)$ $m = \dfrac{59.7-38.1}{60} = 21.6\ mi/hr$

iv. the fourth hour

$P(180,59.7)$
$Q(240,80)$ $m = \dfrac{80-59.7}{60} = 20.3\ mi/hr$

(b) Estimate the cyclist's average velocity (in miles per hour) in the time period $[60, 90]$.

$P(60,18.7)$
$Q(90,23.1)$ $m = \dfrac{23.1-18.7}{90-60} = \dfrac{4.4}{30}\ \dfrac{mi}{min} = 8.8\ mi/hr$

(c) Estimate the cyclist's average velocity (in miles per hour) in the time period $[90, 120]$.

$P(90,23.1)$
$Q(120,38.1)$ $m = \dfrac{38.1-23.1}{30} = \dfrac{15.0}{30}\ \dfrac{mi}{min} = 30\ mi/hr$

(d) Estimate how fast the cyclist was going 1.5 hours into the ride.

$\dfrac{30+8.8}{2} = \dfrac{38.8}{2} = 19.4\ mi/hr$

(e) During what period do you estimate the cyclist was riding the fastest on average?

Between 90min and 120min where cyclist averaged 30 mi/hr

(f) What does any this have to do with secant lines and tangent lines?

a, b, c are slopes of secant lines. d is an estimate of the slope of a tangent line