1. Sketch the graph of an example of a function f that satisfies *all* of the given conditions.

   (a) $f(0) = 2$

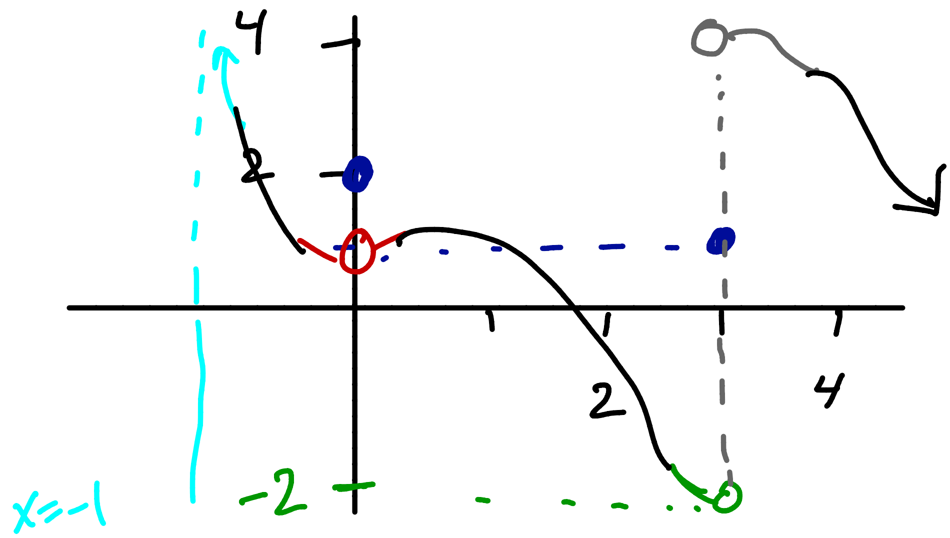
   (b) $f(3) = 1$

   (c) $\lim_{x\to 0} f(x) = 1$

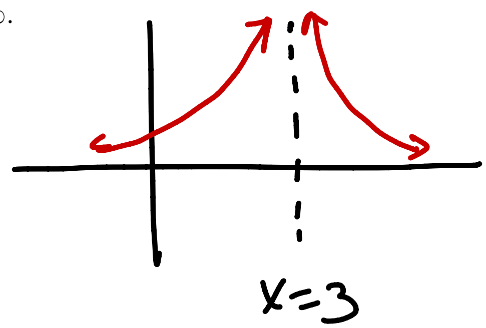
   (d) $\lim_{x\to 3^-} f(x) = -2$

   (e) $\lim_{x\to 3^+} f(x) = 4$

   (f) $\lim_{x\to -1^+} f(x) = \infty$



2. Sketch a graph $f(x)$ such that $\lim_{x\to 3} f(x) = \infty$.



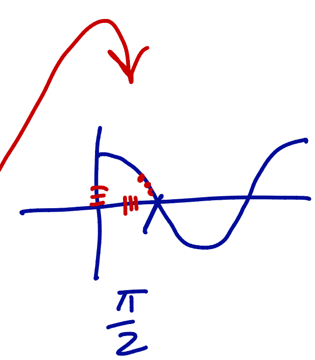
3. (a) Use a calculator and a table of values to determine the limit: $\lim_{x\to 1^-} x\sec(\pi x/2) = \boxed{\infty}$

x	0.5	0.9	0.99	0.9999	0.99999999
$x\sec(\frac{x\pi}{2}) = \frac{x}{\cos(\pi x/2)}$	0.707	5.75	63.0	6365	6.3×10^7

   (b) Use mathematical reasoning to show that your answer in part (a) is correct.

as $x \to 1^-$, $x \to 1$, but $\cos(\frac{\pi x}{2}) \to 0^+$.

So $\frac{1}{\cos(\frac{\pi x}{2})} \to \infty$.



This is a hint that real life rarely offers...

4. Without using a calculator, determine the (infinite) limit. Explain your reasoning.

(a) $\displaystyle\lim_{x\to 3^-} \frac{\sqrt{x}}{x-3} = -\infty$

$x \approx 2.9$

as $x \to 3^-$, $\sqrt{x} \to \sqrt{3}$

$x - 3 \to 0^-$

aside

$\displaystyle\lim_{x\to 3} \frac{\sqrt{3}}{x-3} = \text{DNE}$

(b) $\displaystyle\lim_{x\to 3^+} \frac{\sqrt{x}}{x-3} = \infty$

$x \approx 3.1$

as $x \to 3^+$, $\sqrt{x} \to \sqrt{3}$

$x - 3 \to 0^+$

(c) $\displaystyle\lim_{x\to 3^+} \frac{2-10x}{x-3} = -\infty$

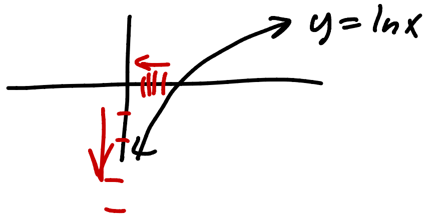
$x \approx 3.1$

as $x \to 3^+$, $2 - 10x \to -28$

$x - 3 \to 0^+$

(d) $\displaystyle\lim_{x\to 3^+} \ln(x-3) = -\infty$

as $x \to 3^+$, $x - 3 \to 0^+$



(e) Why didn't we ask you to find $\displaystyle\lim_{x\to 3^-} \ln(x-3)$?

If $x < 3$, then $x - 3 < 0$. So $\ln(x-3)$ isn't defined.

(f) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x} - \ln(x)\right) = \lim_{x\to 0^+} \left(\frac{1}{x} + (-\ln x)\right) = \infty$

From graphs I see that BOTH head to $+\infty$.
So together they must approach $+\infty$