§2.2 The Limit of a Function

Symbols $\lim_{x \to a} f(x) = L$

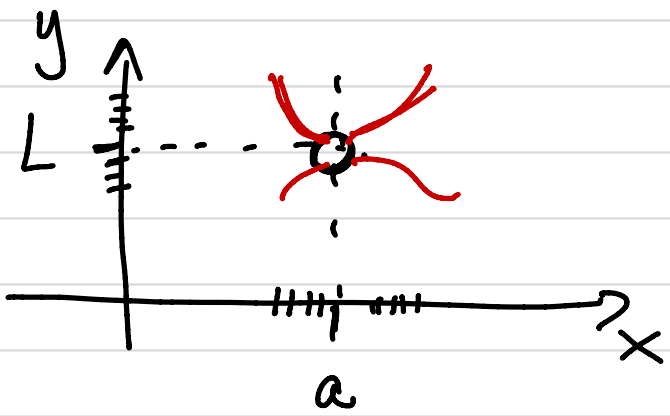
words " the limit of f(x), as x approaches a, is L "

meaning The output of f(x) can be forced arbitrarily close to L by picking x's sufficiently close to a.

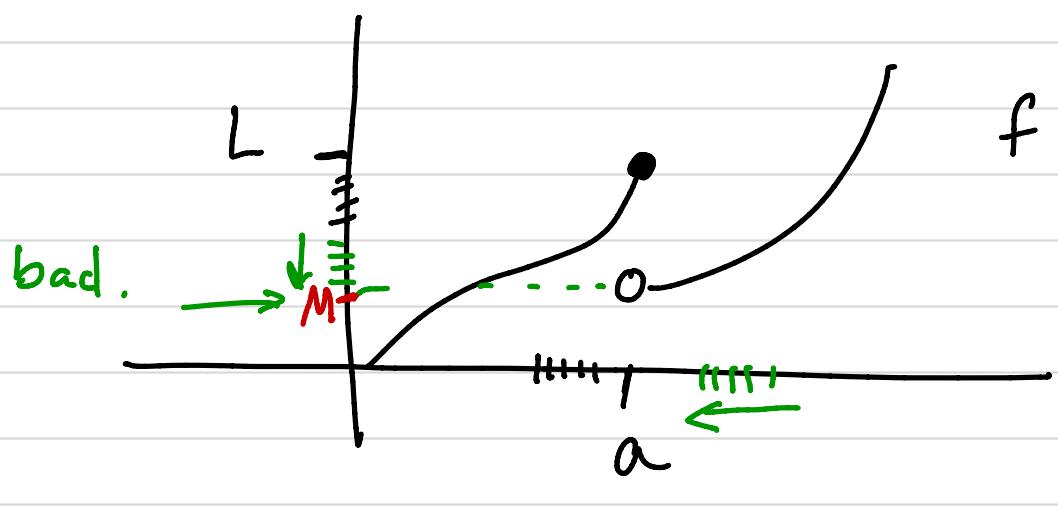
or

as x gets close to a, f(x) gets close to L

Picture



✓



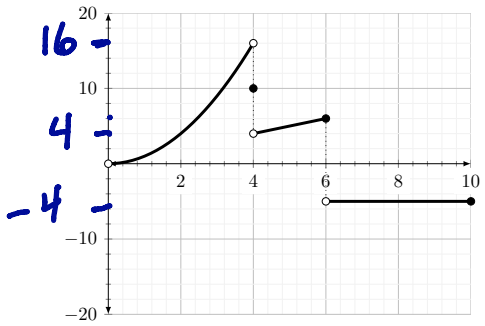
$\lim_{x \to a} f(x)$ does not exist.

Alternatively: $\lim_{x \to a^+} f(x) = M$ right hand limit

$\lim_{x \to a^-} f(x) = L$ left hand limit

- How else can a limit fail to exist? approach infinity, Kooky stuff
  vertical asymptote.
- Does the y-value at $x=a$ affect whether the limit exists? (No)
  ↑ on $y=f(x)$

1. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x\to 4^-} f(x) =$ 16

(b) $\lim_{x\to 4^+} f(x) =$ 4

(c) $\lim_{x\to 4} f(x) =$ DNE

(d) $f(4) =$ 10

(e) $\lim_{x\to 6^-} f(x) =$ 6

(f) $\lim_{x\to 6^+} f(x) =$ −4

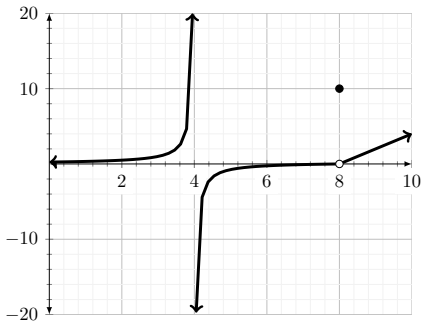
(g) $\lim_{x\to 6} f(x) =$ DNE

(h) $f(6) =$ 6

(i) $\lim_{x\to 8} f(x) =$ −4

(j) $f(8) =$ −4

2. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x\to 4^-} f(x) =$ $+\infty$

(b) $\lim_{x\to 4^+} f(x) =$ $-\infty$

(c) $\lim_{x\to 4} f(x) =$ DNE

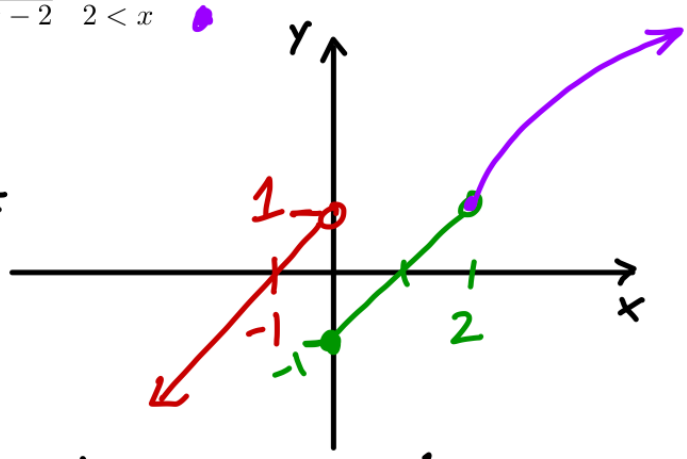
(d) $f(4) =$ DNE

(e) $\lim_{x\to 8} f(x) =$ 0

(f) $f(8) =$ 10

Write the equation of any vertical asymptotes: $x = 4$

3. Evaluate the limits below by graphing $f(x) = \begin{cases} x+1 & x < 0 \\ x-1 & 0 \le x < 2 \\ 1+\sqrt{x-2} & 2 < x \end{cases}$



(a) $\lim_{x \to 0} f(x)$ = DNE

because the limit on left and the limit on right are different

(b) $\lim_{x \to 2} f(x)$ = 1

$\lim_{x \to 2^+} f(x) = 1 = \lim_{x \to 2^-} f(x)$.

(c) For which values a does $\lim_{x \to a} f(x)$ exist? All real numbers except $x=0$.