1. Fill in the blanks below. Assume a and c are fixed constants. (Note that these are all in your text but not in this order.) **Assume** $\lim_{x\to a} f(x)$ **and** $\lim_{x\to a} g(x)$ **exist.**

(a) $\lim_{x\to a} c =$ c ← f(x) is a horizontal line

(b) $\lim_{x\to a} x =$ a ← f(x) is a line.

(c) $\lim_{x\to a} (f(x)+g(x)) = \left[\lim_{x\to a} f(x)\right] + \left[\lim_{x\to a} g(x)\right]$ ← Evaluate each term in limit separately. ☺

i. What do the rules above imply about $\lim_{x\to 12} (x+\pi)$? $= \lim_{x\to 12} x + \lim_{x\to 12} \pi = \boxed{12+\pi}$

(d) $\lim_{x\to a} (f(x)-g(x)) = \left[\lim_{x\to a} f(x)\right] - \left[\lim_{x\to a} g(x)\right]$

(e) $\lim_{x\to a} cf(x) = (c)\left(\lim_{x\to a} f(x)\right)$ ← Take constants out side the limit.

i. What do the rules above imply about $\lim_{x\to 5} 2x+3$? $= 2\left(\lim_{x\to 5} x\right) + \lim_{x\to 5} 3 = 2\cdot 5+3 = 13$

(f) $\lim_{x\to a} f(x)g(x) = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$ ← See ☺ above

(g) $\lim_{x\to a} x^n =$ a^n ← $x^n = (x)(x)(x)\cdots(x)$

(h) $\lim_{x\to a} (f(x))^n = \left(\lim_{x\to a} f(x)\right)^n$ $\quad (f(x))^n = (f(x))(f(x))\cdots(f(x))$

(i) $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} f(x) \Big/ \lim_{x\to a} g(x)$ provided $\lim_{x\to a} g(x) \neq 0$

(j) $\lim_{x\to a} \sqrt[n]{x} =$ $a^{1/n} = \sqrt[n]{a}$

(k) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

2. If $\lim_{x\to\sqrt{2}} f(x) = 8$ and $\lim_{x\to\sqrt{2}} g(x) = e^2$, then evaluate

$$\lim_{x\to\sqrt{2}} \left(\frac{g(x)}{(3-f(x))^2} + 2\sqrt{g(x)} \right)$$

← find limit of each piece separately.

$$= \frac{e^2}{(3-8)^2} + 2\sqrt{e^2} = \frac{e^2}{25} + 2e$$

in effect, we just "plugged in"

3. Use the previous rules to evaluate (a) and explain why you *cannot* use the rules to evaluate (b).

(a) $\lim_{w\to-\frac{1}{2}} \frac{2w+1}{w^3} = \frac{2(-\frac{1}{2})+1}{(-\frac{1}{2})^3} = \frac{0}{-\frac{1}{8}} = 0$

(b) $\lim_{t\to 1} \frac{t^2+t-2}{t^2-1} = \frac{1^2+1-2}{1^2-1} = \frac{0}{0}$ ← Bad! See Rule (i)

4. (One more super-useful rule!) If $f(x) = g(x)$ when $\boxed{x \neq a}$!!, then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$

(provided the limits exist)!

5. Use this rule *and what you know about zeros of polynomials* to evaluate

$$\lim_{t\to 1} \frac{t^2+t-2}{t^2-1} = \lim_{t\to 1} \underbrace{\frac{(t-1)(t+2)}{(t-1)(t+1)}}_{f} = \lim_{t\to 1} \underbrace{\frac{t+2}{t+1}}_{g} = \frac{1+2}{1+1} = \frac{3}{2}$$