Jill's examples

$y = x^2(x+1)$

1. Complete **The Product Rule:** If f and g are differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)] = f' \cdot g + f \cdot g'$$

$y = \frac{x^2}{x+1}$

2. Complete **The Quotient Rule:** If f and g are differentiable, then

order matters

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2} = \frac{g \cdot f' - f \cdot g'}{(g)^2}$$

plausibility:

$y = \frac{5}{x^2}$

3. Find the derivatives for each function below. *Do not use the Product Rule or the Quotient Rule if you don't have to!*

(a) $f(x) = 5x^3e^x$

$$f'(x) = \frac{d}{dx}[5x^3] \cdot e^x + (5x^3)\frac{d}{dx}[e^x]$$
$$= 15x^2e^x + 5x^3e^x$$
$$= 5x^2e^x(3+x)$$

$16x - 4x^2 + 2x^2 - 5$

(b) $f(x) = \frac{2x^2 - 5}{4 - x}$

$$f'(x) = \frac{(4-x)\frac{d}{dx}[2x^2-5] - (2x^2-5) \cdot \frac{d}{dx}[4-x]}{(4-x)^2} = \frac{(4-x)(4x) - (2x^2-5)(-1)}{(4-x)^2} = \frac{-2x^2+16x-5}{(4-x)^2}$$

(c) $f(x) = (1 - x^2)(e^x + x)$

$$f'(x) = (-2x)(e^x+x) + (1-x^2)(e^x+1)$$

(d) $g(x) = \frac{\sqrt{x}}{8}(1 - x\sqrt{x}) = \frac{1}{8} \cdot x^{1/2}\left(1 - x^{3/2}\right) = \frac{1}{8}\left(x^{\frac{1}{2}} - x^2\right)$

$$g'(x) = \frac{1}{8}\left(\frac{1}{2}x^{-1/2} - 2x^1\right) = \frac{1}{16}x^{-1/2} - \frac{1}{4}x$$

(e) $h(x) = \dfrac{10x - x^{3/2}}{4x^2}$ $= \frac{1}{4} \cdot x^{-2}\left(10x - x^{3/2}\right) = \frac{1}{4}\left(10x^{-1} - x^{-1/2}\right)$

$$h'(x) = \frac{1}{4}\left(-10x^{-2} + \frac{1}{2}x^{-3/2}\right)$$

$2x^{1/3} . 3x^{2/3}$

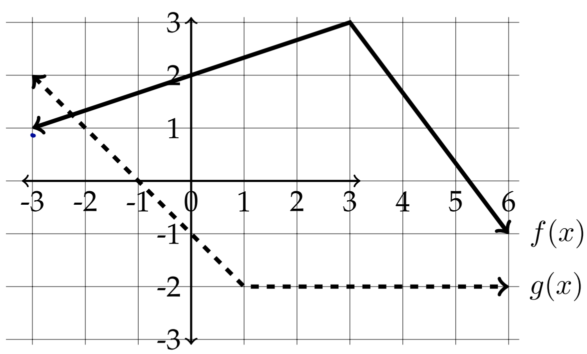
(f) $y = \dfrac{\sqrt[3]{x}}{2x+1}$ $= \dfrac{x^{1/3}}{2x+1}$

$$y' = \frac{(2x+1)\cdot\frac{1}{3}x^{-2/3} - x^{1/3}(2)}{(2x+1)^2} = \frac{2x+1-6x}{3x^{2/3}(2x+1)^2} = \frac{1-4x}{3x^{2/3}(2x+1)^2}$$

(g) $v(t) = \dfrac{2te^t}{t^2+1}$

$$v'(t) = \frac{(t^2+1)(2\cdot e^t + 2te^t) - 2te^t(2t)}{(t^2+1)^2} = \frac{2e^t(t^3 - t^2 + t + 1)}{(t^2+1)^2}$$

4. The graphs of $f(x)$ (shown thick) and the graphs of $g(x)$ (shown dashed) are shown below. If $h(x) = f(x)g(x)$, find $h'(0)$.



$$h'(0) = f'(0)g(0) + f(0)\cdot g'(0)$$
$$= \left(\frac{1}{3}\right)(-1) + (2)(-1)$$
$$= -2 - \frac{1}{3} = -7/3$$

5. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = 2$. Find the following values.

(a) $(f-g)'(5)$

$f'(5) - g'(5)$

$= 6 - 2 = 4$

(b) $(fg)'(5)$

$f(5)g'(5) + f'(5)g(5)$

$= 1\cdot(2) + 6(-3)$

$= -16$

(c) $(g/f)'(5)$

$\dfrac{f\cdot g' - g\cdot f'}{f^2}$

$= \dfrac{1\cdot 2 - (-3)\cdot 6}{1^2} = 20$