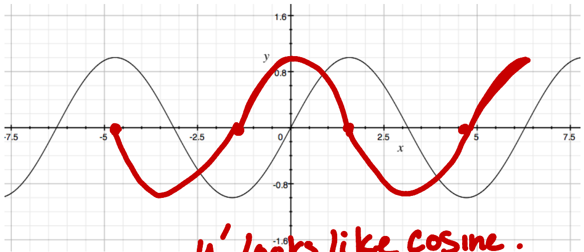
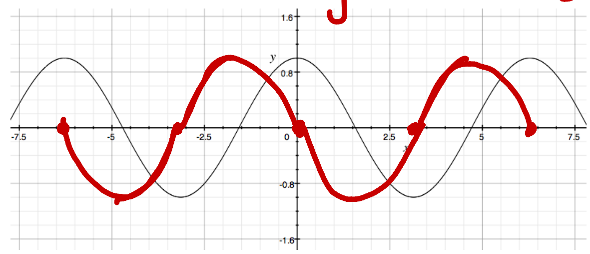
Section 3.3 Derivatives of Trig Functions

1. Use the graphs of $y = \sin x$ and $y = \cos x$ to sketch a graph of y'.





y' looks like cosine.

y' looks like $-$sine

2. Use what we learned in 4. and 5. above to find the derivative of:

(a) $y = 3x^4\cos(x)$

$$y' = 12x^3\cos x - 3x^4\sin x = 3x^3(4\cos x - x\sin x)$$

(b) $y = \tan(x)$ (Use the Quotient Rule.) $= \dfrac{\sin x}{\cos x}$

$$y' = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

3. Fill in the table below.

Derivatives of Trigonometric Functions:

- $\frac{d}{dx}(\sin x) =$ $\cos x$
- $\frac{d}{dx}(\cos x) =$ $-\sin x$
- $\frac{d}{dx}(\tan x) =$ $\sec^2 x$
- $\frac{d}{dx}(\csc x) =$ $-\csc x\cot x$
- $\frac{d}{dx}(\sec x) =$ $\sec x\tan x$
- $\frac{d}{dx}(\cot x) =$ $-\csc^2 x$

4. Find the derivative of $y = \dfrac{\sec x}{1 - x\tan x}$.

$$y' = \frac{(1-x\tan x)(\sec x\tan x) - (\sec x)\left[0 - 1\cdot\tan x - x\sec^2 x\right]}{(1-x\tan x)^2}$$

$$= \frac{\sec x\left[\tan x(1-x\tan x) + \tan x + x\sec^2 x\right]}{(1-x\tan x)^2}$$

5. If $f(\theta) = e^\theta \sin(\theta)$, find $f''(\theta)$. Simplify your answers here.

$$f'(\theta) = e^\theta(\cos\theta) + e^\theta\sin\theta = e^\theta(\cos\theta + \sin\theta)$$

$$f''(\theta) = e^\theta(-\sin\theta + \cos\theta) + e^\theta(\cos\theta + \sin\theta)$$

$$= 2e^\theta\cos\theta$$

6. Find $\dfrac{d}{dt}[t\sin t\cos t]$. $= (t\sin t)\cdot\dfrac{d}{dt}[\cos t] + \dfrac{d}{dt}[t\sin t]\cdot\cos t$

$$= (t\sin t)(-\sin t) + (1\cdot\sin t + t\cos t)\cos t$$

$$= t(\cos^2 t - \sin^2 t) + \sin t\cos t$$

7. An elastic band is hung on a hook and a mass is hung on the lower end of the band. When the mass is pulled down 2 cm past its rest position and then released, it vibrates vertically. The equation of motion is

$$s = 2\cos t + 3\sin t, \quad \text{for } t \geq 0,$$

where s is measured in centimeters and t is measured in seconds. (We are taking the positive direction to be downward.)

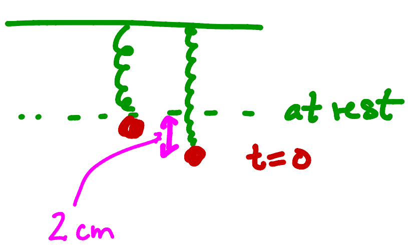
(a) Find $s(0)$, $s'(0)$, and $s''(0)$ including units.

$s(t) = 2\cos t + 3\sin t$; $\quad s(0) = 2$ cm

$s'(t) = -2\sin t + 3\cos t$; $\quad s'(0) = 3$ cm/s

$s''(t) = -2\cos t - 3\sin t$; $\quad s''(0) = -2$ cm/s^2

(b) What do the numbers from part (a) indicate about the mass in the context of the problem?



S(0) confirms the mass is pulled 2 cm below resting

S'(0) tells us the mass wasn't just "let go" but was released with downward velocity of 3 cm/s.

S'' confirms that the spring is pulling up on the mass, causing it to slow down.