

SECTION 3.4 CHAIN RULE (DAY 2)

Evaluate the derivatives.

$$1. H(x) = \sqrt[3]{\frac{4-2x}{5}} = \frac{1}{\sqrt[3]{5}} (4-2x)^{1/3}$$

$$H'(x) = \frac{1}{\sqrt[3]{5}} \cdot \frac{1}{3} (4-2x)^{-2/3} (-2) = \frac{-2}{3\sqrt[3]{5} (4-2x)^{2/3}}$$

$$2. y = e^{\sec \theta}$$

$$y' = (\sec \theta \tan \theta) e^{\sec \theta}$$

$$3. f(x) = \frac{8}{x^2 + \sin(x)} = 8(x^2 + \sin x)^{-1}$$

$$f'(x) = -8(x^2 + \sin x)^{-2} (2x + \cos x) = \frac{-8(2x + \cos x)}{(x^2 + \sin x)^2}$$

$$4. x(t) = \frac{1}{\sqrt{2}} \tan\left(\frac{\pi}{6} - x\right)$$

$$x'(t) = \frac{1}{\sqrt{2}} \cdot \sec^2\left(\frac{\pi}{6} - x\right) (-1) = -\frac{1}{\sqrt{2}} \sec^2\left(\frac{\pi}{6} - x\right)$$

$$5. y = \frac{x e^{-\pi x^2/10}}{100} = \frac{1}{100} (x e^{-\frac{\pi}{10} x^2})$$

$$y' = \frac{1}{100} \left(e^{-\frac{\pi}{10} x^2} + x e^{-\frac{\pi}{10} x^2} \left(-\frac{2\pi}{10} x\right) \right) = \frac{e^{-\frac{\pi}{10} x^2}}{100} \left(1 - \frac{\pi}{5} x^2 \right)$$

$$6. y = \frac{e^2 - x}{5 + \cos(5x)}$$

$$y' = \frac{(5 + \cos(5x))(-1) - (e^2 - x)(-\sin(5x)(5))}{[5 + \cos(5x)]^2} = \frac{5(e^2 - x)\sin(5x) - (5 + \cos(5x))}{(5 + \cos(5x))^2}$$

$$7. y = e^{2t/(1-t)} = e^{2\left(\frac{t}{1-t}\right)}$$

$$y' = e^{2\left(\frac{t}{1-t}\right)} \left(2 \cdot \frac{(1-t)(1) - t(-1)}{(1-t)^2} \right)$$

$$= \left(e^{\frac{2t}{1-t}} \right) \left(\frac{2}{(1-t)^2} \right)$$

$$8. f(x) = \cos^3\left(\frac{8}{1+x^2}\right) = \left(\cos \left[8(1+x^2)^{-1} \right] \right)^3$$

$$f'(x) = 3 \left(\cos \left[8(1+x^2)^{-1} \right] \right)^2 \left(-\sin \left(8(1+x^2)^{-1} \right) \right) \left(-8(1+x^2)^{-2} (2x) \right)$$

$$= \frac{48}{(1+x^2)^2} \sin\left(\frac{8}{1+x^2}\right) \cos^2\left(\frac{8}{1+x^2}\right)$$

$$9. h(x) = (x + (x + \sin(2x))^5)^{1/2}$$

$$h'(x) = \frac{1}{2} \left(x + (x + \sin(2x))^5 \right)^{-1/2} \left(1 + 5(x + \sin(2x))^4 (1 + 2\cos(2x)) \right)$$

$$10. F(x) = (2re^{rx} + n)^p \text{ (Assume } r, n, \text{ and } p \text{ are fixed constants.)}$$

$$F'(x) = p (2re^{rx} + n)^{p-1} (2r \cdot re^{rx})$$

$$= 2pr^2 e^{rx} (2re^{rx} + n)$$