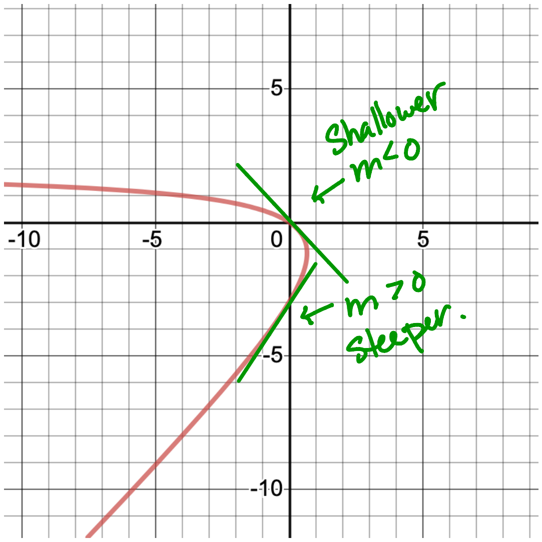
1. Find $\frac{dy}{dx}$ for $2x + 3y = xy - y^2$ and find the equations of tangents to the graph when $x = 0$. Use the graph below as an aid and to determine the plausibility of your answers.



$$2x + 3y = xy - y^2$$

$$2 + 3y' = 1\cdot y + x\cdot y' - 2yy'$$

$$2 - y = (x - 2y - 3)y'$$

$$\boxed{y' = \frac{2-y}{x-2y-3}}$$

at $(0,0)$, $y' = -2/3$

at $(0,-3)$, $y' = \frac{5}{3}$

lines:

$y = \frac{-2}{3}x$

$y = \frac{5}{3}x - 3$

2. Find $\frac{da}{db}$ for $a^3 \sin(3b) = a^2 - b^2$

$$\left(3a^2 \cdot \frac{da}{db}\right)\sin(3b) + a^3\cos(3b)(3) = 2a\frac{da}{db} - 2b$$

$$\left(3a^2\sin(3b) - 2a\right)\frac{da}{db} = -3a^3\cos(3b) - 2b$$

$$\boxed{\frac{da}{db} = \frac{-3a^3\cos(3b) - 2b}{3a^2\sin(3b) - 2a}}$$

3. Find $\frac{dy}{dx}$ for $e^{xy} = x + y + 1$

$$\left(e^{xy}\right)\frac{d}{dx}[xy] = 1 + \frac{dy}{dx} + 0$$

$$e^{xy}\left(1\cdot y + x\cdot\frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

$$ye^{xy} + xe^{xy}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\left(xe^{xy} - 1\right)\frac{dy}{dx} = 1 - ye^{xy}$$

$$\boxed{\frac{dy}{dx} = \frac{1 - ye^{xy}}{xe^{xy} - 1}}$$

4. You are going to derive the formula for the derivative of inverse tangent the way we found the derivative of inverse sine in class.

   (a) Find dy/dx for the expression $x = \tan(y)$.

   $$1 = \sec^2(y) \cdot \frac{dy}{dx}, \quad \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

   (b) Use the identity $1 + \tan^2(\theta) = \sec^2(\theta)$ to rewrite you answer in part (a) and *write your dy/dx in terms of x only*.

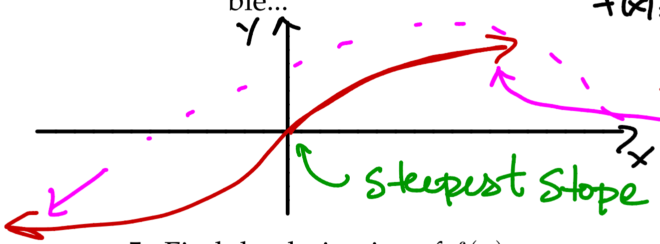
   $$\frac{dy}{dx} = \frac{1}{1+\tan^2(y)} = \frac{1}{1+x^2}$$

   because $x = \tan y$

   (c) Now fill in the blank $\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$

   (d) Use your knowledge of the *graph* of $f(x) = \arctan(x)$ to decide if your answer seems plausible...

   $f(x) = \arctan x$



   ← slope always positive.

   shallower slope as $x \to \pm\infty$

   steepest slope here where $x = 0$

5. Find the derivative of $f(x) = x \arctan x$.

   $$f'(x) = \boxed{1 \cdot \arctan x + \frac{x}{1+x^2}}$$

6. Find the derivative of $f(x) = \arctan(4 - x^2)$.

   $$f'(x) = \frac{1}{1+(4-x^2)^2} \cdot \frac{d}{dx}(4-x^2) = \boxed{\frac{-2x}{1+(4-x^2)^2}}$$