1. Fill in the derivative rules below:

$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\arccos(x)] = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$

$\frac{d}{dx}[\log_b(x)] = \frac{1}{(\ln b)x}$

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

2. Find the derivative of each function below:

(a) $y = \ln(x^5) = 5\ln x$

$y' = 5 \cdot \frac{1}{x} = \frac{5}{x}$

(b) $y = (\ln x)^5$

$y' = 5(\ln x)^4 \cdot \frac{1}{x}$

$= \frac{5(\ln x)^4}{x}$

(c) $y = \ln(5x) = \ln 5 + \ln x$

$y' = \frac{1}{5x} \cdot 5 = \frac{1}{x}$

$y' = 0 + \frac{1}{x} = \frac{1}{x}$

3. Find the derivative of each function below:

(a) $f(x) = x^2 \log_2(5x^3 + x)$

$f'(x) = 2x\log_2(5x^3+x) + x^2 \cdot \frac{1}{(\ln 2)(5x^3+x)} \cdot (15x^2+1)$

$= 2x\log_2(5x^3+x) + \frac{x^2(15x^2+1)}{(\ln 2)(5x^3+x)}$

(b) $g(x) = \ln(x^2 \tan^2 x) = 2\ln x + 2\ln(\tan x)$

$g'(x) = \frac{2}{x} + \frac{2\sec^2 x}{\tan x}$

Pay attention to where the product is...

4. Find $\frac{dy}{dx}$ for $y = \ln\sqrt{\frac{x+\sin x}{x^2-e^x}}$. $= \frac{1}{2}\ln(x+\sin x) - \frac{1}{2}\ln(x^2-e^x)$

$$\frac{dy}{dx} = \frac{1+\cos x}{2(x+\sin x)} - \frac{2x-e^x}{2(x^2-e^x)}$$

5. Find y' for each of the following:

(a) $y = \ln|x|$ $\boxed{y' = \frac{1}{x}}$

aside:

$$\ln|x| = \begin{cases} \ln x & x>0 \\ \ln(-x) & x<0 \end{cases}$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln(-x)] = \frac{-1}{-x} = \frac{1}{x}$$

The same formula in either case.

(b) $y = \frac{e^{-x}\sin x}{\sqrt{1-x^2}}$

trick: use implicit diff

$$\ln y = -x + \ln(\sin x) - \frac{1}{2}\ln(1-x^2)$$

$$\frac{1}{y}\frac{dy}{dx} = -1 + \frac{\cos x}{\sin x} + \frac{2x}{2(1-x^2)};\quad \text{So, } \frac{dy}{dx} = \left(\frac{e^{-x}\sin x}{\sqrt{1-x^2}}\right)\left(-1 + \cot x + \frac{x}{1-x^2}\right)$$

(c) $y = x^{\sqrt[3]{x}}$

trick: implicit differentiation

$$\ln y = x^{1/3}\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}x^{-2/3}\ln x + x^{1/3}\cdot\frac{1}{x}$$

$$\text{So, } \frac{dy}{dx} = x^{\sqrt[3]{x}}\left(\frac{\ln x}{3x^{2/3}} + \frac{1}{x^{2/3}}\right) = \frac{x^{\sqrt[3]{x}}(\ln x + 3)}{3x^{2/3}}$$