

SECTION 3.7 RATES OF CHANGE IN THE NATURAL AND SOCIAL SCIENCES

1. A particle moves according to the law of motion $s(t) = 2 - 15t + 4t^2 - \frac{1}{3}t^3$, for $t \geq 0$, where t is measured in seconds and s is measured in feet.

(a) Find the velocity at time t .

$$v(t) = s'(t) = -15 + 8t - t^2$$

(b) What is the velocity after 1 second?

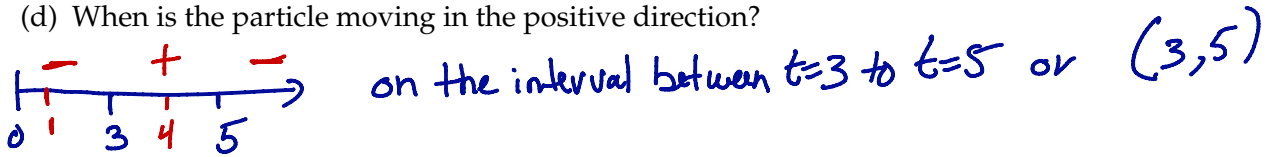
$$v(1) = s'(1) = -15 + 8 - 1 = \underline{\underline{-8 \text{ ft/sec}}}$$

(c) When is the particle at rest? Find t when $v=0$.

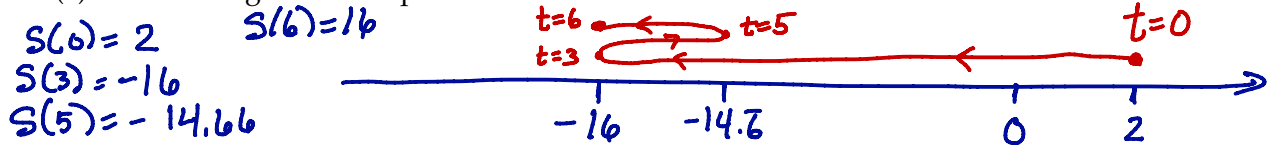
$$0 = -15 + 8t - t^2 = -(t-3)(t-5)$$

$$t = 3, 5$$

(d) When is the particle moving in the positive direction?



(e) Draw a diagram of the particle from $t = 0$ to $t = 6$.



(f) Find the displacement of the particle during the first 6 seconds.

$$-18 \qquad 1\frac{1}{3}$$

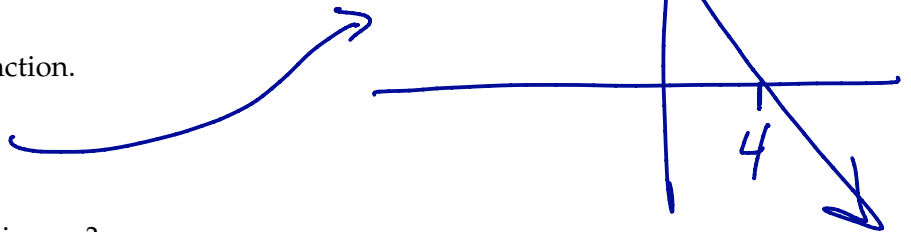
(g) Find the total distance traveled by the particle during the first 6 seconds.

$$18 + 2\left(\frac{4}{3}\right) = 18\frac{8}{3}$$

(h) Find the acceleration of the particle.

$$a(t) = v'(t) = s''(t) = 8 - 2t$$

(i) Graph the acceleration function.

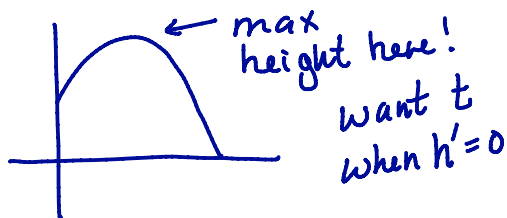


(j) When is the particle speeding up?

$$3 \leq t \leq 4 \text{ and } t \geq 5$$

2. The height (in meters) of a projectile shot vertically upward from a point 10 meters above ground level with an initial velocity of 20 meters per second is $h = 10 + 20t - 4.9t^2$.

(a) When does the projectile reach its maximum height?



$$h'(t) = 20 - 9.8t = 0$$

$$t = \frac{20}{9.8} = \frac{10}{4.9} \text{ sec } (\approx 2.04 \text{ s})$$

(b) What is its maximum height?

$$h\left(\frac{10}{4.9}\right) = 10 + 20\left(\frac{10}{4.9}\right) - 4.9\left(\frac{10}{4.9}\right)^2 = 30.408 \text{ m}$$

(c) When does the projectile hit the ground? Find t when $h=0$.

$$\text{Solve } 0 = 10 + 20t - 4.9t^2$$

$$t = \frac{10}{4.9} \left(10 \pm \sqrt{149} \right) \text{ . Chose positive : } t \approx 4.532 \text{ s}$$

answer

(d) What what velocity does it hit the ground? $v(4.532) \approx -24.4136 \text{ m/s}$

3. A tank holds 1000 gallons of a fluid, which drains from the bottom of the tank in 30 minutes. The function below give the volume of fluid remaining in the tank after t minutes:

$$V(t) = 1000 \left(1 - \frac{1}{30}t \right)^2 \text{ for } 0 \leq t \leq 30$$

Find the rate at which the fluid is draining from the tank after 10 minutes. When is the fluid flowing the fastest? Slowest?

$$V'(t) = 2000 \left(1 - \frac{1}{30}t \right) \left(-\frac{1}{30} \right) = -\frac{200}{3} \left(1 - \frac{1}{30}t \right)$$

$$V'(10) = -\frac{200}{3} \left(1 - \frac{1}{3} \right) = -\frac{200}{3} \left(\frac{2}{3} \right) = -\frac{400}{9} \approx 44.44 \text{ gal/min}$$

$$|V'(t)| \text{ largest when } t=0: \frac{200}{3}$$

$$|V'(t)| \text{ smallest when } t=30: 0$$