1. A population of flies is assumed to grow with constant relative growth rate. Suppose there are 100 flies after the second day and 400 flies after the fourth day.

   (a) What is the relative growth rate?

$\frac{1}{P} \cdot \frac{dP}{dt} = k$

Use $P(t) = Ce^{kt}$.

$100 = Ce^{2k}$
$400 = Ce^{4k}$

So $\left(100e^{-2k} = 400e^{-4k}\right)\left(\frac{e^{4k}}{100}\right)$

$e^{2k} = 4$

$2k = \ln 4$

$k = \frac{1}{2}\ln 4 = \ln 2 \approx 0.693147$

   (b) What was the initial size of the population?

$100 = Ce^{(2\ln 2)}$

So $C = 100/e^{2\ln 2} = 25$

Check

$P(t) = 25e^{(\ln 2)t}$

$P(2) = 100$

$P(4) = 400$

   (c) Find an expression fo the number of flies after t days.

$P(t) = 25e^{(\ln 2)t}$

   (d) When will the population of flies be 10,000?

$10{,}000 = 25e^{(\ln 2)t}$

$400 = e^{(\ln 2)t}$

$\ln(400) = (\ln 2)t$

$t = \frac{\ln(400)}{\ln(2)} \approx 8.6438$ days

2. Radium-226 has a half-life of 1590 years. Assume one starts with 50 mg of Radium-226. Find an expression for the amount m of Radium-226 in terms of t.

- $m(t) = m_0 e^{kt}$
- $m_0 = 50\text{ mg}$
- when $t = 1590$, $m = 50/2 = 25\text{ mg}$
- Find k: $25 = 50 e^{(k)(1590)}$

$$\frac{1}{2} = e^{1590k}$$

$$-\ln(2) = \ln\left(\tfrac{1}{2}\right) = 1590k$$

$$k = -\ln(2)/1590 \approx 0.0004359$$

$$m(t) = 50 e^{\left(\frac{-\ln 2}{1590}\right)t}$$

3. When a cup of coffee is poured it is 95 degrees Celsius. After 10 minutes the coffee has cooled to 80 degrees Celsius. If the surrounding temperature is 20 degrees Celsius, find an expression for the temperature of the coffee T in terms of time t.

Newton's Law of Cooling

$$\frac{dT}{dt} = k(T - T_s)$$

when $t=0$, $T = 95$

when $t=10$, $T = 80$

$T_s = 20$

$\boxed{\text{Let } y = T - T_s.}$

So $y(0) = 95 - 20 = \underbrace{75}_{y_0}$

$y(10) = 80 - 20 = 60$

Now

$$y(t) = y_0 e^{kt}$$

$$y = 75 e^{kt}$$

$$60 = 75 e^{10k}$$

$$\frac{60}{75} = \frac{4}{5} = e^{10k}$$

$$\frac{\ln(4/5)}{10} = k$$

$$y(t) = 75 e^{\frac{\ln(4/5)}{10}t}$$

$T(t) = T_s + y(t)$

$$= 20 + 75 e^{\frac{\ln(4/5)}{10}t}$$