Review Day 4: Inverse Function, Exponential Functions, & Logarithmic Functions

1. In your own words, explain what it means for $f^{-1}(x)$ to be the *inverse* of $f(x)$? You might try explaining it using graphs, algebra, or numerical calculations.

① $a \to \boxed{f(x)} \to b$

$b \to \boxed{f^{-1}(x)} \to a$

② f^{-1} undoes f

③ Given $f(x)$, find $f^{-1}(x)$ by "switching" x and y

④ the graph of f^{-1} is the graph of f reflected over line $y=x$.

2. Without doing a bunch of algebra, find $f^{-1}(x)$ for each function below:

   (a) $f(x) = 2x$

   $y = 2x$

   $x = 2y$ or $y = \frac{1}{2}$

   $f^{-1}(x) = \frac{1}{2}x$

   (b) $f(x) = x^3$

   $y = x^3$

   $x = y^3$ or $y = \sqrt[3]{x}$

   $f^{-1}(x) = \sqrt[3]{x}$

3. Without explicitly finding a formula for $f^{-1}(x)$, find $f^{-1}(1)$ for each function below:

   (a) $f(x) = x - 20$

   $f^{-1}(1) = \quad$ means

   $1 \to \boxed{f^{-1}} \to \quad$

   So $\quad \to \boxed{f} \to 1$

   what is this? $x=21$. ANS: $f^{-1}(1)=21$

   (b)

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2.0
$f(x)$	20	10	5	3	2.5	2	1.5	1	0.25

   ANS: $f^{-1}(1) = 1.75$

4. Explain why the directions "Find $f^{-1}(1)$" don't make sense for the following examples:

   (a) $f(x) = x^2 - 3$

   problem: $f(2)=1$ and $f(-2)=1$.

   So $f^{-1}(1)$ could be 2 or -2

   Not a function.

   It's not clear How to undo $f(x)$...

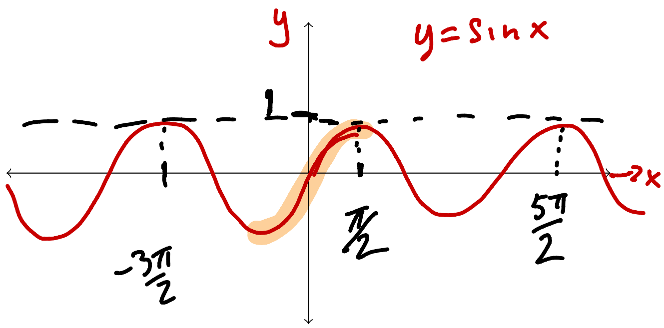
   (b)

x	0	1	2	3	4	5	6	7	8
$f(x)$	-3	1	5	8	6	2	3	1	0

   So which one is it?

   $f^{-1}(1)=1$ or $f^{-1}(1)=7$??

5. Give a not-too-big rough sketch of $f(x) = \sin x$ and ask yourself whether or not it makes since to be asked to find $\sin^{-1}(1)$. (Recall that $\sin^{-1}(1)$ could be written $\arcsin(1)$ or $\operatorname{invsin}(1)$.)



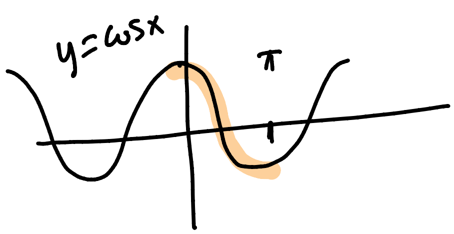
$\sin^{-1}(1) =$

$\sin(\quad) = 1$

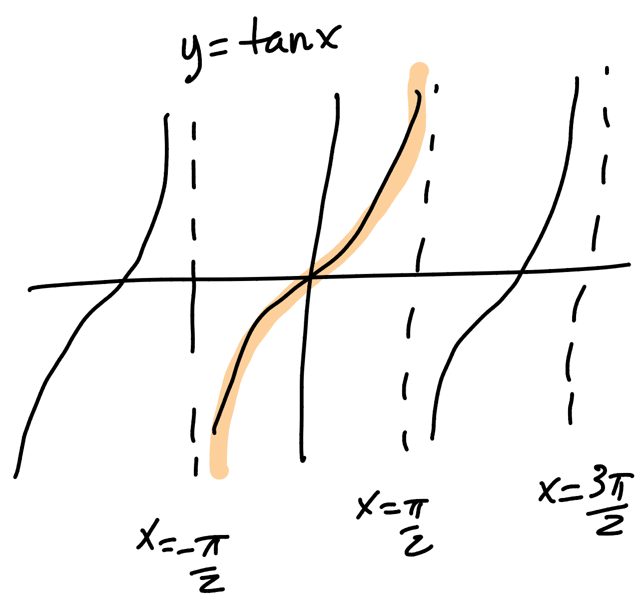
$= -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2} \ldots$

which one?

again: $\arcsin(-1) = -\frac{\pi}{2}$



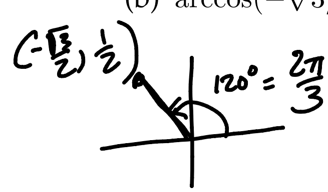
$\arccos(-1) = \pi$



6. Evaluate the following:

(a) $\arcsin(1) = \pi/2$

(b) $\arccos(-\sqrt{3}/2) = 2\pi/3$



$y = \cos x$

$\boxed{x = \cos y}$

or

$y = \arccos x$

$y = \arccos(-\sqrt{3}/2)$

or

$-\frac{\sqrt{3}}{2} = \cos y$

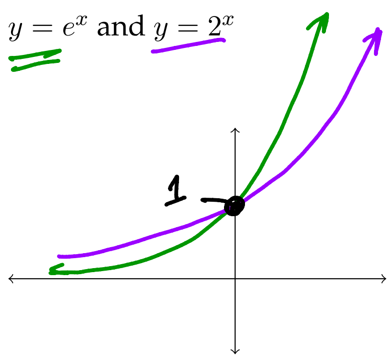
(c) $\arctan(1) = -\frac{\pi}{4}$

(d) $\arcsin(10) =$ DNE

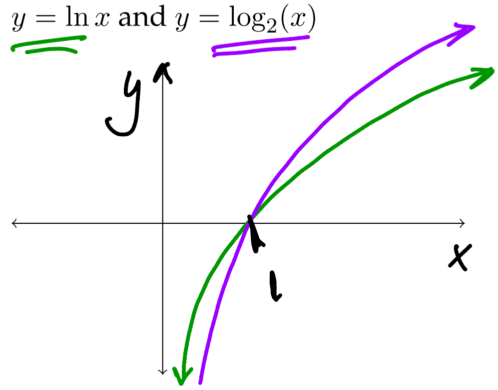
arcsine will only input values between -1 and 1 (!!)

7. On the axes below, sketch:

(a) $y = e^x$ and $y = 2^x$



(b) $y = \ln x$ and $y = \log_2(x)$



- Simplify $2^3 \cdot 2^4 = 2^7$ because $2^3 \cdot 2^4 = (2\cdot2\cdot2)(2\cdot2\cdot2\cdot2) = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 = 2^7$
- what is e^{-1}? e^2? $e^{1/2}$?

  $= \frac{1}{e}$ $\quad e \cdot e$ $\quad \sqrt{e}$

$\log_4 64 = y$ OR $4^y = 64$ So $y = 3$

- what is $\log_4 2$? $\log_4 64$?

$\log_4 2 = \boxed{?}$ or $4^{\boxed{?}} = 2$

$? = \frac{1}{2}$

- $y = \log_4 x$ is the inverse of $y = 4^x$. OR

$\boxed{y = \log_4 x}$ is equivalent to $\boxed{x = 4^y}$

8. Find the exact value of each expression.

(a) $\log_2 16 = 4$

because $2^4 = 16$

(b) $e^{\ln 5} = 5$

because e^x and $\ln x$ are inverses.

So $e^{\ln x} = x$ (also $\ln(e^x) = x$)

5. Solve each equation below for x.

(a) $10 = 2e^{x+1}$

$5 = e^{x+1}$

$\ln 5 = x+1$

$x = (\ln 5) - 1$

(b) $\ln(x^2 - 1) = 1$

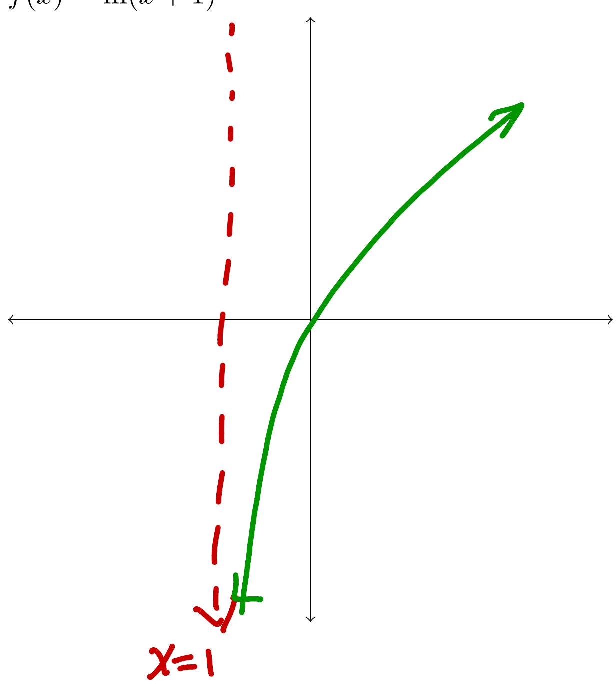
$x^2 - 1 = e^1$

$x^2 = e + 1$

$x = \pm\sqrt{e+1}$

6. Sketch each function. Include domain, range, intercepts and asymptotes.

(a) $f(x) = \ln(x + 1)$



(b) $f(x) = -\ln x$ reflect about x-axis