SECTION 2-6 (DAY 1)

Evaluate the limits below. You may use graphs or numerical calculation to confirm your answer, but your *formal* answer must be **algebraic**.

1. $\lim_{x\to-\infty} \frac{3x^2+4x}{2x^4+7}$ $\cdot \frac{1/x^4}{1/x^4} = \lim_{x\to-\infty} \frac{\frac{3}{x^2}+\frac{4}{x^3}}{2+7/x^4} = \frac{0+0}{2+0} = \frac{0}{2} = 0$

2. $\lim_{x\to\infty} \frac{5x^{5/2}-8x^2+1}{2x^2+7}$ $\cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{\overset{\nearrow\infty}{5\sqrt{x}} \overset{\nearrow -8}{-8} + \overset{\nearrow 0}{\frac{1}{x^2}}}{\underset{\searrow 2}{2} + \underset{\searrow 0}{7/x^2}} = \infty$

3. $\lim_{x\to\infty} \frac{2e^x}{8-\sqrt{5}e^x}$ $\cdot \frac{e^{-x}}{e^{-x}} = \lim_{x\to\infty} \frac{2}{8e^{-x}-\sqrt{5}} = \frac{2}{0-\sqrt{5}} = \frac{-2}{\sqrt{5}}$

4. $\lim_{x\to-\infty} \frac{2e^x}{8-\sqrt{5}e^x}$ $= \frac{2\cdot 0}{8-\sqrt{5}\cdot 0} = \frac{0}{8} = 0$

Hint: $x^3 = \sqrt{x^6}$ provided $x > 0$.

5. $\lim_{x\to\infty} \frac{\sqrt{3x^6-x}}{x^3+1}$ $\cdot \frac{x^{-3}}{x^{-3}} = \lim_{x\to\infty} \frac{\sqrt{3-\frac{1}{x^5}}}{1+x^{-3}} = \frac{\sqrt{3-0}}{1+0} = \sqrt{3}$

6. $\lim_{x\to-\infty} \frac{\sqrt{3x^6-x}}{x^3+1}$ $\cdot \frac{x^{-3}}{x^{-3}} = \lim_{x\to-\infty} \frac{-\sqrt{3-x^{-5}}}{1+x^{-3}} = -\sqrt{3}$

7. $\lim_{x\to-\infty} e^{\arctan x}$ $= e^{-\pi/2}$

(use $\lim_{x\to-\infty} \arctan x = \frac{-\pi}{2}$)

8. $\lim_{x\to\infty} [\ln(2+3x) - \ln(1+x)]$ $= \lim_{x\to\infty} \ln\left(\frac{2+3x}{1+x}\right) = \ln\left[\lim_{x\to\infty} \frac{2+3x}{1+x}\right] = \ln\left[\lim_{x\to\infty} \frac{\frac{2}{x}+3}{\frac{1}{x}+1}\right] = \ln\left(\frac{0+3}{0+1}\right) = \ln 3$

$\infty - \infty$

9. $\lim_{x\to\infty} (\sqrt{x^2+x} - x) \cdot \left(\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}\right) = \lim_{x\to\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x} \cdot \frac{x^{-1}}{x^{-1}}$

$\infty - \infty$

$= \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$

10. $\lim_{x\to-\infty} \sqrt[3]{x} - x^3$

$-\infty + \infty$

$= \lim_{x\to-\infty} \sqrt[3]{x}\left(1 - x^{\frac{26}{3}}\right) = (-\infty)(-\infty) = \infty$

$\ominus \cdot \ominus$

11. $\lim_{x\to\infty} e^{-2x} + \cos x$ $= \lim_{x\to\infty}\left(\underbrace{\frac{1}{e^{2x}}}_{0} + \underbrace{\cos x}_{DNE}\right) = DNE$

12. $\lim_{x\to\infty} e^{-2x}\cos x$ (Hint: Use the Squeeze Theorem.)

$\lim_{x\to\infty} \frac{1}{e^{2x}} = 0 = \lim_{x\to\infty} \frac{-1}{e^{2x}}$ and $\frac{-1}{e^{2x}} \le \frac{\cos x}{e^{2x}} \le \frac{1}{e^{2x}}$; So $\lim_{x\to\infty} \frac{\cos x}{e^{2x}} = 0$

13. Find all vertical and horizontal asymptotes in the graph of the function $g(s) = \dfrac{\sqrt{3s^2+1}}{2s+1}$.

* See solutions to Friday's sheet!