$P(a, 3a^2)$, $Q(a+h, 3(a+h)^2)$

1. Find the slope of the tangent line to $f(x) = 3x^2$ at $x = a$ by taking the limit of the slopes of secant lines. When you are done, check whether or not your solutions seems plausible!

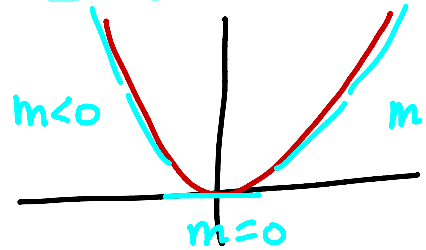
$$m_{tan} = \lim_{h\to 0} m_{sec} = \lim_{h\to 0} \frac{3(a+h)^2 - 3a^2}{(a+h) - (a)} = \lim_{h\to 0} \frac{3a^2+6ah+3h^2-3a^2}{h}$$

$$= \lim_{h\to 0} \frac{6ah+3h^2}{h} = \lim_{h\to 0} 6a+3h = 6a+3\cdot 0 = \boxed{6a}$$

plausible? when $a<0$, $m_{tan} = 6a < 0$

$a=0$, $m_{tan} = 6\cdot 0 = 0$

$a>0$, $m_{tan} = 6a > 0$



2. Write the equation of the line tangent to the graph of $f(x) = 3x^2$ at $x = \frac{1}{2}$.

$m_{tan} = 6a$

when $x = \frac{1}{2}$, $m_{tan} = 6(\frac{1}{2}) = 3$

point $(\frac{1}{2}, 3(\frac{1}{2})^2) = (\frac{1}{2}, \frac{3}{4})$

tangent line: $y - \frac{3}{4} = 3(x - \frac{1}{2})$ or $\boxed{y = 3(x-\frac{1}{2}) + \frac{3}{4}}$

See this!

3. The derivative of a function f at $x = a$ is $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} m_{sec}$

4. Assume the tangent line to the graph of $y = f(x)$ at $x = \sqrt{2}$ is $y = \frac{4x-1}{3}$. Determine:

(a) $f(\sqrt{2}) = \boxed{\frac{4\sqrt{2}-1}{3}}$

tangent line and $f(x)$ share a point at $x = \sqrt{2}$.

So $\frac{4(\sqrt{2})-1}{3}$ ← plug $x=\sqrt{2}$ into line

(b) $f'(\sqrt{2}) = \boxed{\frac{4}{3}}$ ← Pick off coeff. of x in line.

Slope of $f(x)$ is the same as slope of tangent line at $x=\sqrt{2}$.

5. The height in meters of an object is given by the function $s(t) = \frac{2}{t+1}$ where t is measured in seconds.

(a) Find $s'(a)$ using the definition in # 3 on this sheet.

(b) Determine the units of $s'(a)$.

(c) Find and interpret in the context of the problem the meaning of $s'(1)$.

(a) $$s'(a) = \lim_{h\to 0} \frac{1}{h}\left(\frac{2}{a+h+1} - \frac{2}{a+1}\right) = \lim_{h\to 0} \frac{1}{h}\left(\frac{2(a+1)-2(a+h+1)}{(a+h+1)(a+1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2a+2-2a-2h-2}{(a+h+1)(a+1)}\right) = \lim_{h\to 0} \frac{1}{h}\left(\frac{-2h}{(a+h+1)(a+1)}\right)$$

$$= \lim_{h\to 0}\left(\frac{-2}{(a+h+1)(a+1)}\right) = \boxed{\frac{-2}{(a+1)^2}}$$

(b) units of $s' = \frac{\text{units of } s}{\text{units of } t} = m/sec$

(c) $s'(1) = \frac{-2}{(1+1)^2} = \frac{-2}{4} = -\frac{1}{2}$ m/s. At the instant 1 second has passed, the height of the object is decreasing at a rate of $-\frac{1}{2}$ m/s.

6. Let $f(x) = \sqrt{90 - x}$

(a) Find $f'(a)$ using the definition in # 3 on this sheet.

(b) If f is measured in degrees celsius and x is measured in minutes, determine the units of $f'(a)$.

(c) Find and interpret $s'(0)$.

(a) $$f'(a) = \lim_{h\to 0} \frac{\sqrt{90-(a+h)} - \sqrt{90-a}}{h} \cdot \frac{\left(\sqrt{90-(a+h)} + \sqrt{90-a}\right)}{\left(\sqrt{90-(a+h)} + \sqrt{90-a}\right)} = \lim_{h\to 0} \frac{90-(a+h)-(90-a)}{h\left(\sqrt{90-(a+h)}+\sqrt{90-a}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{h\left(\sqrt{90-(a+h)} + \sqrt{90-a}\right)} = \lim_{h\to 0} \frac{-1}{\sqrt{90-(a+h)}+\sqrt{90-a}} = \frac{-1}{2\sqrt{90-a}}$$

(b) units of $f'(a)$ = $\frac{\text{units of } f(x)}{\text{units of } x} = \frac{\text{degrees Celsius}}{\text{minutes}} = {}^\circ C/min$

(c) $s'(0) = \frac{-1}{2\sqrt{90}} = \frac{-1}{6\sqrt{10}}$ °C/min. When time starts, the temperature is decreasing at a rate of $-1/6\sqrt{10}$ degrees Celsius per minute.

Bonus: Find & interpret $f(0)$.