We have been proving stuff about derivatives for weeks now....

EXAMPLE: If $f(x) = 3x^2$, then $f'(x) =$

1. Find the slope of the tangent line to $f(x) = 3x^2$ at $x = a$ by taking the limit of the slopes of secant lines. When you are done, check whether or not your solutions seems plausible!

$$m_{tan} = \lim_{h\to 0} m_{sec} = \lim_{h\to 0} \frac{3(a+h)^2 - 3a^2}{(a+h)-(a)} = \lim_{h\to 0} \frac{3a^2+6ah+3h^2-3a^2}{h}$$

$$= \lim_{h\to 0} \frac{6ah+3h^2}{h} = \lim_{h\to 0} 6a+3h = 6a+3\cdot 0 = 6a$$

plausible? when $a<0$, $m_{tan} = 6a < 0$

EXAMPLE: If $f(x) = \sqrt{90 - x}$, then $f'(x) =$

$$\text{(a) } f'(a) = \lim_{h\to 0} \frac{\sqrt{90-(a+h)} - \sqrt{90-a}}{h} \cdot \frac{\left(\sqrt{90-(a+h)} + \sqrt{90-a}\right)}{\left(\sqrt{90-(a+h)} + \sqrt{90-a}\right)} = \lim_{h\to 0} \frac{90-(a+h)-(90-a)}{h\left(\sqrt{90-(a+h)} + \sqrt{90-a}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{h\left(\sqrt{90-(a+h)} + \sqrt{90-a}\right)} = \lim_{h\to 0} \frac{-1}{\sqrt{90-(a+h)} + \sqrt{90-a}} = \frac{-1}{2\sqrt{90-a}}$$

EXAMPLE: If $f(x) = 2x - \frac{2}{x}$, then $f'(x) =$

2. Let $f(x) = 2x - \frac{2}{x}$.

(a) Use the definition to find the derivative of $f'(a)$.

$$f'(a) = \lim_{h\to 0} \frac{\left[2(a+h) - \frac{2}{a+h}\right] - \left[2a - \frac{2}{a}\right]}{h}$$

$$= \lim_{h\to 0} \frac{2a+2h-2a + \frac{2}{a} - \frac{2}{a+h}}{h} = \lim_{h\to 0} \left[2 + \frac{1}{h}\left(\frac{2a+2h-2a}{(a)(a+h)}\right)\right]$$

$$= \lim_{h\to 0} \left[2 + \frac{2}{(a)(a+h)}\right] = 2 + \frac{2}{a^2}$$

1. Fill in the derivative rules. Then practice using each rule to find y' if y is given.

   (a) $\frac{d}{dx}[c] =$ ______________________ $y = 5$ $y' =$ ______________________

   (b) $\frac{d}{dx}[x^n] =$ ______________________ $y = x^{50}$ $y' =$ ______________________

   (c) $\frac{d}{dx}[c\,f(x)] =$ ______________________ $y = 3x^2$ $y' =$ ______________________

   (d) $\frac{d}{dx}[f(x) + g(x)] =$ ______________________ $y = 5x^6 + x^7$ $y' =$ ______________________

   (e) $\frac{d}{dx}[f(x) - g(x)] =$ ______________________ $y = 6x^3 - x$ $y' =$ ______________________

   (f) $\frac{d}{dx}[e^x] =$ ______________________ $y = \frac{1}{2}e^x$ $y' =$ ______________________

2. Compute derivatives of the following functions using derivative rules. **Do not simplify your answers.** (If you already know what these are, DO NOT USE THE PRODUCT RULE, THE QUOTIENT RULE OR THE CHAIN RULE. If you don't know what they are, presumably you won't be using them either!)

   (a) $f(x) = (x - 2)(2x + 3)$

   (b) $g(x) = \frac{x^2}{2} - \frac{2}{x^2} + \frac{1}{\sqrt{2}}$

(c) $f(t) = \sqrt{t} - e^t + t^{0.3}$

(d) $f(x) = \dfrac{x^2 + x - 1}{\sqrt{x}}$

(e) $V(r) = \dfrac{4}{3}\pi r^3$

(f) $f(x) = e^{x-3}$

(g) $H(r) = a^2 r^2 + br + c$

3. At what point(s) on the curve $y = 3x + x^3$ is the tangent to the curve parallel to the line $y = 6x - 5$?