SECTION 2-2: THE LIMIT OF A FUNCTION

Read Section 2.2. Work the embedded problems.

why? $f(2) = \frac{0}{0}$??

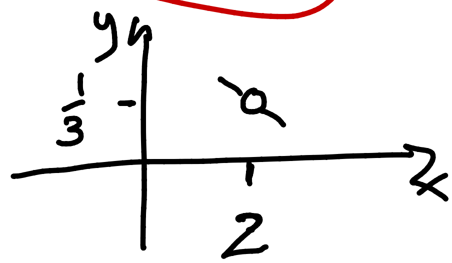
1. EXAMPLE 1: What does the function $f(x) = \frac{x-2}{x^2-x-2}$ look like around $x = 2$?

table of values

closer to 2 → ← closer to 2

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	0.34483	0.33445	0.33344	DNE	0.33322	0.33223	0.32258

Based on table, I'd say y-values are close to $\frac{1}{3}$ when x-values are close to 2 (both sides)



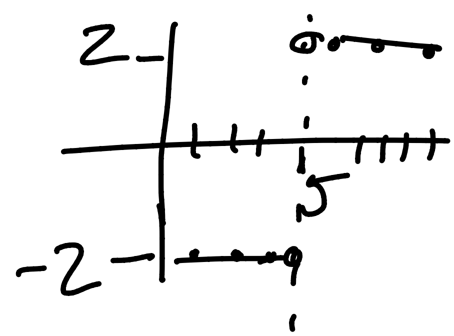
$$\lim_{x\to 2} \frac{x-2}{x^2-x-2} = \frac{1}{3}$$

You do 2. EXAMPLE 2: What does the function $f(x) = \frac{2|x-5|}{(x-5)}$ look like around $x = 5$?

x	4.9	4.99	4.9999	5	5.0001	5.01	5.1
f(x)	-2	-2	-2		2	2	2

$\lim_{x\to 5} \frac{2|x-5|}{x-5} = \text{DNE}$. But,

$\lim_{x\to 5^-} f(x) = -2$, $\lim_{x\to 5^+} f(x) = 2$



For the 2-sided limit to exist, we need the one-sided limits to be =.

As x gets close to 5, f(x) gets close to 2 or -2 DEPENDING on which side!!

* Note : We found a limit on fri when calculating m_{tan}

3. DEFINITION: two-sided limit

Say: "the limit of $f(x)$, as x approaches a is L"

Write: $\lim_{x\to a} f(x) = L$

It means:

on both sides

As x gets closer & closer to a, y=f(x) gets closer and closer to L.

4. DEFINITION: one-sided limits

- Say: "the limit of $f(x)$, as x approaches a *on the left* is L"

  Write: $\lim_{x\to a^-} f(x) = L$

  It means:

  As x gets closer to a from the left (#'s smaller than a), y=f(x) gets closer to L

- Say: "the limit of $f(x)$, as x approaches a *on the right* is L"

  Write: $\lim_{x\to a^+} f(x) = L$

  It means:

  As x gets closer to a from right (#'s larger than a), y=f(x) gets closer to L.

5. EXAMPLE 3: What does the function $f(x) = \frac{8-x}{(x-2)^2}$ look like around $x = 2$?

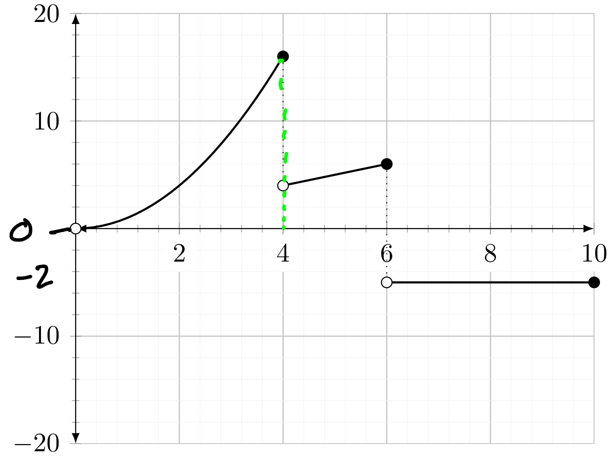
x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	610	60,100	6,001,000	DNE	5,999,000	5,9900	590

As x gets close to 2, f(x) goes to +∞. (y-values get larger without any bound.)

6. DEFINITION: infinite limits $\lim_{x\to a} f(x) = +\infty$, $\lim_{x\to a} f(x) = -\infty$

There are one-sided infinite limits, too!

7. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x\to 4^-} f(x) =$ 16

(b) $\lim_{x\to 4^+} f(x) =$ 4

not equal! So

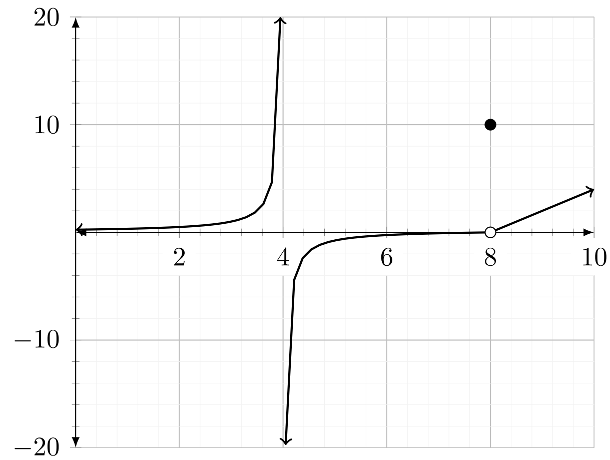
(c) $\lim_{x\to 4} f(x) =$ DNE

(d) $f(4) =$ 16

(e) $\lim_{x\to 8} f(x) =$ −2

(f) $f(8) =$ −2

8. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x\to 4^-} f(x) =$ $+\infty$

(b) $\lim_{x\to 4^+} f(x) =$ $-\infty$

(c) $\lim_{x\to 4} f(x) =$ DNE

(d) $f(4) =$ DNE

(e) $\lim_{x\to 8} f(x) =$ 0

(f) $f(8) =$ 10

Write the equation of any vertical asymptote: $x=4$

9. What is the relationship between limits and vertical asymptotes?

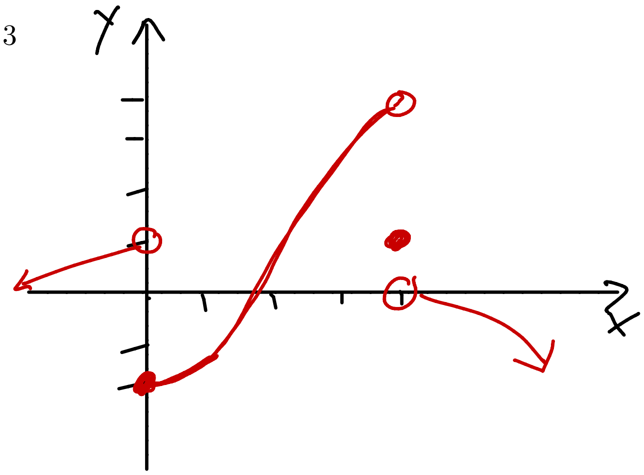
If $\lim_{x\to a \text{ or } x\to a^+ \text{ or } x\to a^-} f(x) = +\infty \text{ or } -\infty$, then $x=a$ is an asymptote.

10. Sketch the graph of an function that satisfies *all* of the given conditions.

$\lim_{x\to 0^-} f(x) = 1$ $\quad \lim_{x\to 0^+} f(x) = -2$ $\quad \lim_{x\to 4^-} f(x) = 3$

$\lim_{x\to 4^+} f(x) = 0$ $\quad f(0) = -2$ ✓ $\quad f(4) = 1$ ✓

(0,-2) (4,1)



11. Some General Principles

(a) $\lim_{x\to 0^-} \frac{1}{x} = -\infty$

(b) $\lim_{x\to 0^+} \frac{1}{x} = +\infty$

(c) $\lim_{x\to 0} \frac{1}{x} =$ DNE

(d) $\lim_{x\to 0^-} \frac{1}{x^2} = +\infty$

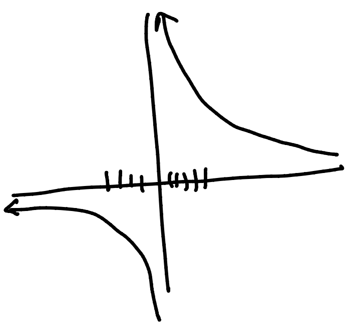
(e) $\lim_{x\to 0^+} \frac{1}{x^2} = +\infty$

(f) $\lim_{x\to 0} \frac{1}{x^2} = +\infty$

(g) $\lim_{x\to a^-} \frac{1}{x-a} = -\infty$

(h) $\lim_{x\to a^+} \frac{1}{x-a} = +\infty$

(i) $\lim_{x\to a} \frac{1}{x-a} =$ DNE



None of these limits exist.
But some don't exist in coherent ways.

Caveat: Very kooky things can happen.