

## SECTION 2-2: THE LIMIT OF A FUNCTION

Read Section 2.2. Work the embedded problems.

1. EXAMPLE 1: What does the function  $f(x) = \frac{x-2}{x^2-x-2}$  look like around  $x = 2$ ?

2. EXAMPLE 2: What does the function  $f(x) = \frac{2|x-5|}{(x-5)}$  look like around  $x = 5$ ?

3. **DEFINITION:** two-sided limit

Say: "the limit of  $f(x)$ , as  $x$  approaches  $a$  is  $L$ "

Write:

It means:

4. **DEFINITION:** one-sided limits

- Say: "the limit of  $f(x)$ , as  $x$  approaches  $a$  on the left is  $L$ "

Write:

It means:

- Say: "the limit of  $f(x)$ , as  $x$  approaches  $a$  on the right is  $L$ "

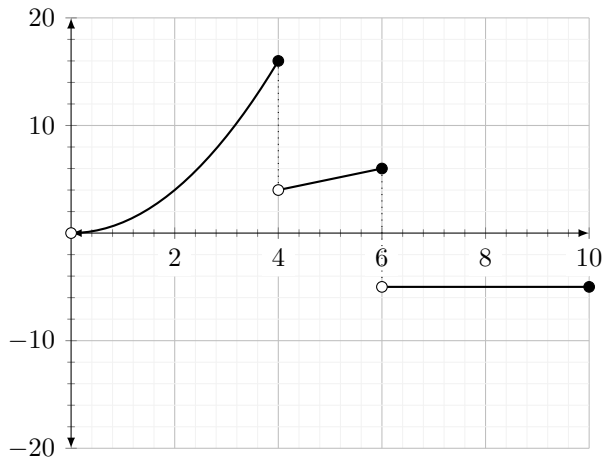
Write:

It means:

5. **EXAMPLE 3:** What does the function  $f(x) = \frac{8-x}{(x-2)^2}$  look like around  $x = 2$ ?

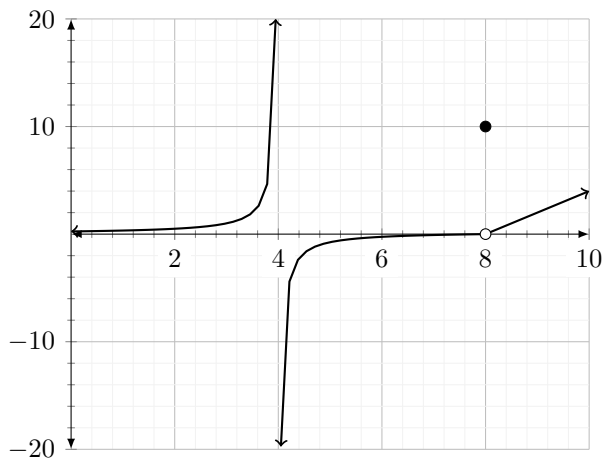
6. **DEFINITION:** infinite limits

7. The function  $g(x)$  is graphed below. Use the graph to fill in the blanks.



- (a)  $\lim_{x \rightarrow 4^-} f(x) = \underline{\hspace{2cm}}$
- (b)  $\lim_{x \rightarrow 4^+} f(x) = \underline{\hspace{2cm}}$
- (c)  $\lim_{x \rightarrow 4} f(x) = \underline{\hspace{2cm}}$
- (d)  $f(4) = \underline{\hspace{2cm}}$
- (e)  $\lim_{x \rightarrow 8} f(x) = \underline{\hspace{2cm}}$
- (f)  $f(8) = \underline{\hspace{2cm}}$

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Write the equation of any vertical asymptote:

9. What is the relationship between limits and vertical asymptotes?

10. Sketch the graph of a function that satisfies *all* of the given conditions.

$$\lim_{x \rightarrow 0^-} f(x) = 1 \quad \lim_{x \rightarrow 0^+} f(x) = -2 \quad \lim_{x \rightarrow 4^-} f(x) = 3$$

$$\lim_{x \rightarrow 4^+} f(x) = 0 \quad f(0) = -2 \quad f(4) = 1$$

11. Some General Principles

(a)  $\lim_{x \rightarrow 0^-} \frac{1}{x} =$

(b)  $\lim_{x \rightarrow 0^+} \frac{1}{x} =$

(c)  $\lim_{x \rightarrow 0} \frac{1}{x} =$

(d)  $\lim_{x \rightarrow 0^-} \frac{1}{x^2} =$

(e)  $\lim_{x \rightarrow 0^+} \frac{1}{x^2} =$

(f)  $\lim_{x \rightarrow 0} \frac{1}{x^2} =$

(g)  $\lim_{x \rightarrow a^-} \frac{1}{x - a} =$

(h)  $\lim_{x \rightarrow a^+} \frac{1}{x - a} =$

(i)  $\lim_{x \rightarrow a} \frac{1}{x - a} =$