Read Section 2.3. Work the embedded problems.

The goal for every problem below is to (a) correctly evaluate the limit, (b) write the mathematics correctly, and (c) articulate *why* you can use the technique you used in part (a).

1. $\lim_{x \to \sqrt{2}} 5x - \sqrt{8x^2 - 1} = 5(\sqrt{2}) - \sqrt{8(\sqrt{2})^2 - 1} = \boxed{5\sqrt{2} - \sqrt{15}}$

① ②

Notes: ① method was substitution. This works b/c no zero in denominator.

② The "$\lim_{x \to \sqrt{2}}$" is no longer written b/c when we substituted in we took the limit

2. $\lim_{t \to 0} \frac{e^{2t} - 1}{1 + \sin(\pi t)} = \frac{e^{2\cdot 0} - 1}{1 + \sin(\pi \cdot 0)} = \frac{e^0 - 1}{1 + \sin(0)} = \frac{0}{1} = 0$

Same Notes as #1 above.

3. $\lim_{x \to -1} \frac{x^2 + 8x + 7}{2x^2 + 3x + 1} = \frac{(-1)^2 + 8(-1) + 7}{2(-1)^2 + 3(-1) + 1} = \frac{8-8}{3-3} = \frac{0}{0}$ ① This tells us substitution will *not* work.

② Try factor and cancel.

$$\lim_{x \to -1} \frac{x^2+8x+7}{2x^2+3x+1} = \lim_{x \to -1} \frac{(x+1)(x+7)}{(x+1)(2x+1)} \overset{③}{=} \lim_{x \to -1} \frac{x+7}{2x+1} = \frac{-1+7}{2(-1)+1} = \frac{6}{-1} = -6$$

③ Why is this fair?
b/c the limit doesn't care about $x = -1$.

4. $\lim\limits_{x\to 5^-} \dfrac{x+1}{5x-x^2} = \dfrac{5+1}{5\cdot 5 - 5^2} = \dfrac{6}{0}$

① This tells us substitution won't work. B/c numerator is not zero, we realize the limit is infinite.

② Do some algebra to figure out the sign.

$$\lim_{x\to 5^-} \frac{x+1}{5x-x^2} = \lim_{x\to 5^-} \underbrace{\left(\frac{x+1}{x}\right)}_{\frac{6}{5}}\underbrace{\left(\frac{1}{5-x}\right)}_{+\infty} = +\infty$$

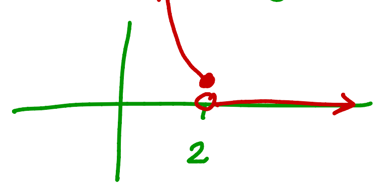
5. $\lim\limits_{x\to -10} \dfrac{2x+g(x)}{\pi f(x)}$ assuming that $\lim\limits_{x\to -10} g(x) = \dfrac{1}{2}$ and $\lim\limits_{x\to -10} f(x) = 1$

$$\lim_{x\to -10} \frac{2x+g(x)}{\pi\, f(x)} = \frac{2(-10)+\frac{1}{2}}{\pi\cdot 1} = \frac{19.5}{\pi}$$

why? limit laws again

$$= \frac{\lim\limits_{x\to -10} 2x + \lim\limits_{x\to -10} g(x)}{\pi \lim\limits_{x\to -10} f(x)}$$

Aside: graph?



6. The last two problems reference the function $f(x) = \begin{cases} \frac{1}{2x} & \text{if } 0 < x \le 2 \\ 0 & \text{if } 2 < x \end{cases}$

(a) Explain. $\lim\limits_{x\to 2} f(x) = \text{DNE}$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{1}{2x} = \frac{1}{2\cdot 2} = \frac{1}{4}$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 0 = 0$$

Since the one-sided limits are not equal, the two-sided limit does not exist.

(b) $\lim\limits_{x\to 2^+} e^{f(x)} = e^{\lim\limits_{x\to 2^+} f(x)} = e^0 = 1$