

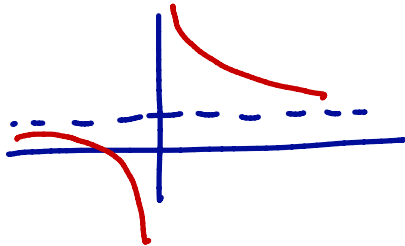
SECTION 2-4: CONTINUITY

Read Section 2.4. Work the embedded problems.

1. Determine the point(s), if any, at which each function is discontinuous. Classify any discontinuity as jump, removable, infinite, or other.

(a) $g(x) = x^{-1} + 1 = \frac{1}{x} + 1$

discontinuity at $x=0$
infinite



(b) $h(x) = \frac{x+2}{x^2-4} = \frac{x+2}{(x+2)(x-2)}$

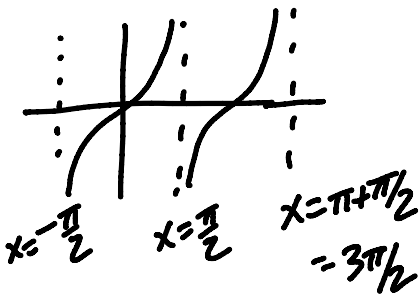
$h(x)$ is discontinuous at $x=2$ and $x=-2$

$x=2$: infinite

$x=-2$: removable

(c) $f(\theta) = \tan(\theta)$

f is discontinuous at $x = \dots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$
all are infinite discontinuities



2. Find the value(s) of k that makes the function continuous over the given interval.

$$f(x) = \begin{cases} e^{kx} & \text{if } 0 \leq x < 4 \\ 2x + 1 & \text{if } 4 \leq x \leq 10 \end{cases}$$

$$f(4) = 2 \cdot 4 + 1 = 9$$

$$\text{We need } \lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} e^{kx} = e^{4k} = 9$$

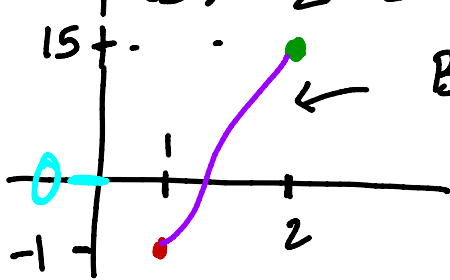
$$\text{So } 4k = \ln 9 \text{ or } \boxed{k = \frac{1}{4} \ln 9}$$

3. Use the Intermediate Value Theorem to show that the equation $x^4 + x - 3 = 0$ must have a solution in the interval from $x = 1$ to $x = 2$.

$f(x) = x^4 + x - 3$ is a polynomial. So, it is continuous for all x -values.

• $f(1) = 1 + 1 - 3 = -1 < 0$

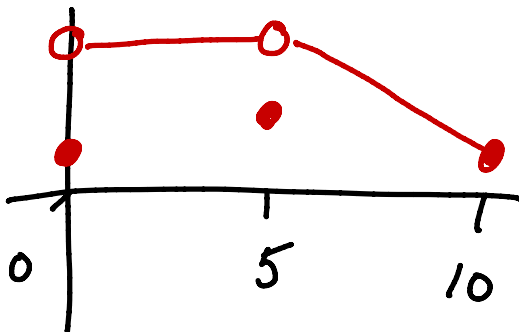
• $f(2) = 2^4 + 2 - 3 = 15 > 0$



Because $f(x)$ is continuous and 0 is between -1 and 15 , IVThm implies there is some c in $(1, 2)$ so that $f(c) = 0$

4. Sketch the graph of a function $f(x)$ with the following properties:

- (a) the domain of $f(x)$ is the interval $[0, 10]$.
 (b) $f(x)$ is continuous except at $x = 0$ and $x = 5$.



OR

