Read Section 3.1. Work the embedded problems.

1. Definition of the Derivative (version 1)

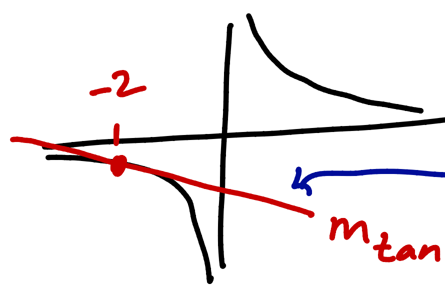
$$f'(a) = m_{tan} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

2. Definition of the Derivative (version 2)

$$f'(a) = m_{tan} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

3. In the problems below, let $f(x) = \frac{1}{x}$.

   (a) Using a *rough* sketch of $f(x)$ make a rough estimate of the slope of the tangent to $f(x)$ when $x = -2$.



looks like slope is negative + not too steep

   (b) Using the first version of the difference quotient, find $m_{\tan}$ when $a=-2$

$$f'(-2) = m_{tan} = \lim_{x\to -2} \frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2} \frac{\frac{1}{x}+\frac{1}{+2}}{x+2} = \lim_{x\to -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2}$$

$$= \lim_{x\to -2}\left(\frac{1}{x+2}\right)\left(\frac{x+2}{2x}\right) = \lim_{x\to -2}\frac{1}{2x} = \frac{1}{-4} = -\frac{1}{4}$$

   (c) Using the second version of the difference quotient, find $m_{\tan}$

$$f'(-2) = \lim_{h\to 0}\frac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0}\frac{\frac{1}{-2+h}+\frac{1}{2}}{h}$$

$$= \lim_{h\to 0}\left(\frac{1}{h}\right)\left(\frac{2+(-2+h)}{(-2+h)(2)}\right) = \lim_{h\to 0}\frac{h}{2(h-2)h} = \lim_{h\to 0}\frac{1}{2(h-2)} = \frac{-1}{4}$$

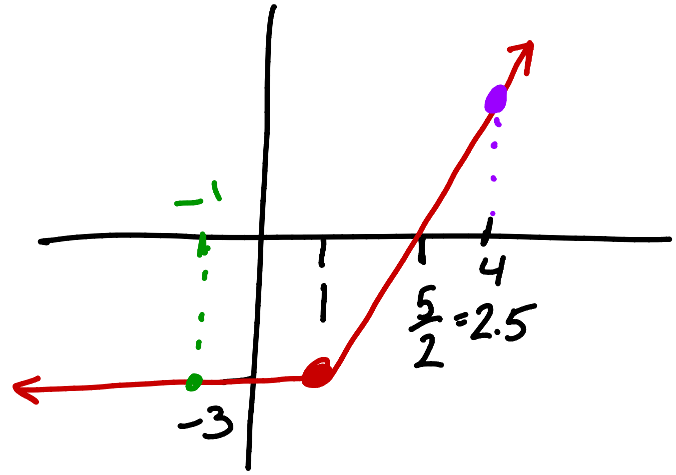
   (d) Write the equation of the line tangent to $f(x)$ when $x = -2$. (Plausible?)

point $\left(-2, -\frac{1}{2}\right)$

$m = -\frac{1}{4}$

line: $y + \frac{1}{2} = -\frac{1}{4}(x+2)$ or $y = -\frac{1}{4}x - 1$ (Looks roughly Correct)

4. Graph the function $G(t) = \begin{cases} -3 & x \le 1 \\ 2x-5 & 1 < x \end{cases}$.



(a) Use the graph to determine $G'(-1)$ and $G'(4)$

- $G'(-1) = 0$
- $G'(4) = 2$

(b) Explain – using the definition – why $G'(1)$ fails to exist.

The definition is a two-sided limit. But we can see that on the right, all secant lines have a slope of $\underline{2}$ but on the left, all secant lines have a slope of $\underline{0}$.

5. A rock is dropped from a height of 100 feet. Its height above ground at time t seconds later is given by $s(t) = -16t^2 + 100$.

(a) Find and interpret $s(0)$ and $s(1)$.

$s(0) = 100$ ft. (When time starts, the rock is dropped from 100ft)

$s(1) = 84$ft (One second after the rock is dropped, it is 84 ft above the ground. Or, it has dropped 16 feet.)

(b) Find and interpret $s'(1)$.

Assume you are told that $s'(1) = -32$ what are its units? Interpret.

units: feet/second

Interpretation: One second after the rock is dropped, the rock is falling at a rate of 32 ft per second.