Read Section 3.3. Work the embedded problems.

1. Review (aka mini-quiz)

   (a) Fill in the following rules – from memory if possible!

   i. $\frac{d}{dx}[f(x)\,g(x)] =$ $f' \cdot g + f \cdot g'$

   ii. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ $\frac{g \cdot f' - f \cdot g'}{(g)^2}$

   (b) Find the derivative of each of the following. Use whatever rule you choose. Simplify if you have time.

   i. $H(x) = \frac{1}{3x}(8 + x^2)$

   (simplify first)

   $H(x) = \frac{1}{3}(8x^{-1} + x)$

   $H'(x) = \frac{1}{3}(-8x^{-2} + 1)$

   $= \frac{-8}{3}x^{-2} + \frac{1}{3}$

   Product Rule

   $H(x) = \frac{1}{3}x^{-1}(8 + x^2)$

   $H'(x) = -\frac{1}{3}x^{-2}(8+x^2) + \frac{1}{3}x^{-1}(2x)$

   $= -\frac{8}{3}x^{-2} - \frac{1}{3} + \frac{2}{3} = -\frac{8}{3}x^{-2} + \frac{1}{3}$

   ii. $G(x) = \frac{3x}{8+x^2}$

   $G'(x) = \frac{(8+x^2)(3) - 3x(2x)}{(8+x^2)^2} = \frac{24 + 3x^2 - 6x^2}{(8+x^2)^2}$

   $= \frac{24 - 3x^2}{(8+x^2)^2} = \frac{3(6-x^2)}{(8+x^2)^2}$

2. Determine the point (or points) where the graph $f(x) = x^3$ has a slope of **3** Write the equation of the tangent line at this point (or points). On the same axes, sketch $f(x)$ and the tangent line(s).

$f(x) = x^3$

Slope $= m = f'(x) = 3x^2$

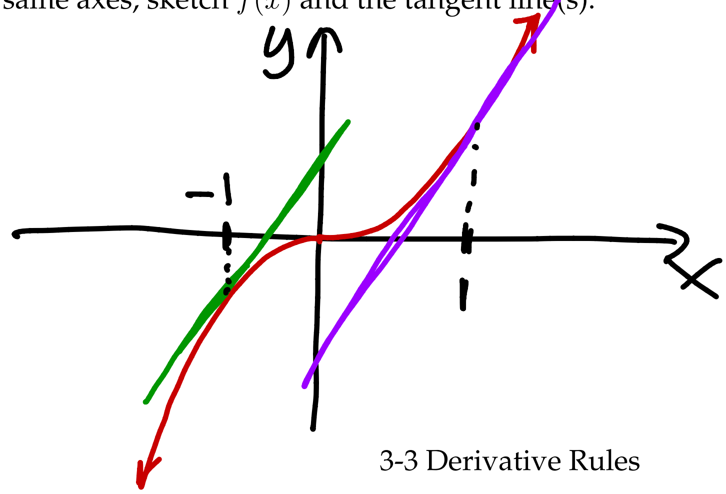
We want $m = 3$.

Set $3x^2 = 3$ or $x = \pm 1$

$P_1(-1,-1)$ $P_2(1,1)$

line 1: $y+1 = 3(x+1)$ or $y = 3x+2$

line 2: $y-1 = 3(x-1)$ or $y = 3x-2$



3. The concentration of an antibiotic in the bloodstream t hours after being injected is given by $C(t) = \dfrac{2t^2+t}{t^3+50}$ where C is measured in milligrams per liter of blood.

   (a) Find $C(0)$ and $C(10)$ and explain (in complete sentences, including units) what these numbers mean in the context of the problem.

   $C(0) = 0,\ C(10) = \dfrac{210}{1050} = 0.2$

   Initially (when $t=0$), there is no antibiotic in the bloodstream. Ten hours after injection, the concentration is 0.2 mg/l

   (b) Find $C'(t)$. (Yes. This will be challenging/painful. I put the simplified answer at the bottom of this page so you can check your answer!)

   $$C'(t) = \frac{(t^3+50)(4t+1) - (2t^2+t)(3t^2)}{(t^3+50)^2} = \frac{-2(t^4+t^3-100t-25)}{(t^3+50)^2}$$

   (c) It is the case that $C'(0) = 0.02$ and $C'(10) = -0.018$. Explain (in a complete sentence or sentences) what these numbers mean. **Include units.**

   Initially, (when $t=0$) the concentration of drug is increasing at a rate of 0.02 mg/l each hour. Ten hours after the drug was administered, the concentration of drug is decreasing at a rate of 0.018 mg/l each hour.

   (d) Briefly describe what seems to be occurring as the number of hours increases.

   Initially the concentration of drug increases, then it starts to decrease. Eventually one would expect the concentration to return to 0.

4. An ant walking along a sidewalk has traveled $s(t) = t^4 - 2t^2$ inches in t minutes. Find the acceleration of the ant (with units) when the velocity of the ant is 0. for $0 \le t \le 5$

   $V = s'(t) = 4t^3 - 4t = 4t(t^2-1) = 0$

   when $t = 0, \pm 1$. (not -1 not in domain)

   $a = v' = s''(t) = 12t^2 - 4$

   at $t=0$, $a(0) = -4$ in/min²

   at $t=1$, $a(1) = 8$ in/min²

5. Bonus Problem: Find the point on the graph of $f(x) = x^3$ such that the tangent line at that point has an x intercept of 6.

$C'(t) = \frac{-2(t^4+t^3-100t-25)}{(t^3+50)^2}$

Bonus Solution:

$f(x) = x^3$ at $x=a$, $P(a, f(a)) = (a, a^3)$

$f'(x) = 3x^2$ slope at $x=a$ is $m = f'(a) = 3a^2$

line through P w/ slope m is:

$$y - a^3 = 3a^2(x-a)$$

or

$$y = 3a^2x - 3a^3 + a^3 = 3a^2x - 2a^3$$

We want $-2a^3 = 6$ or $a^3 = -3$ or $a = -\sqrt[3]{3}$

Answer: The point is: $(-\sqrt[3]{3}, -3)$