Read Section 3.5. Work the embedded problems.

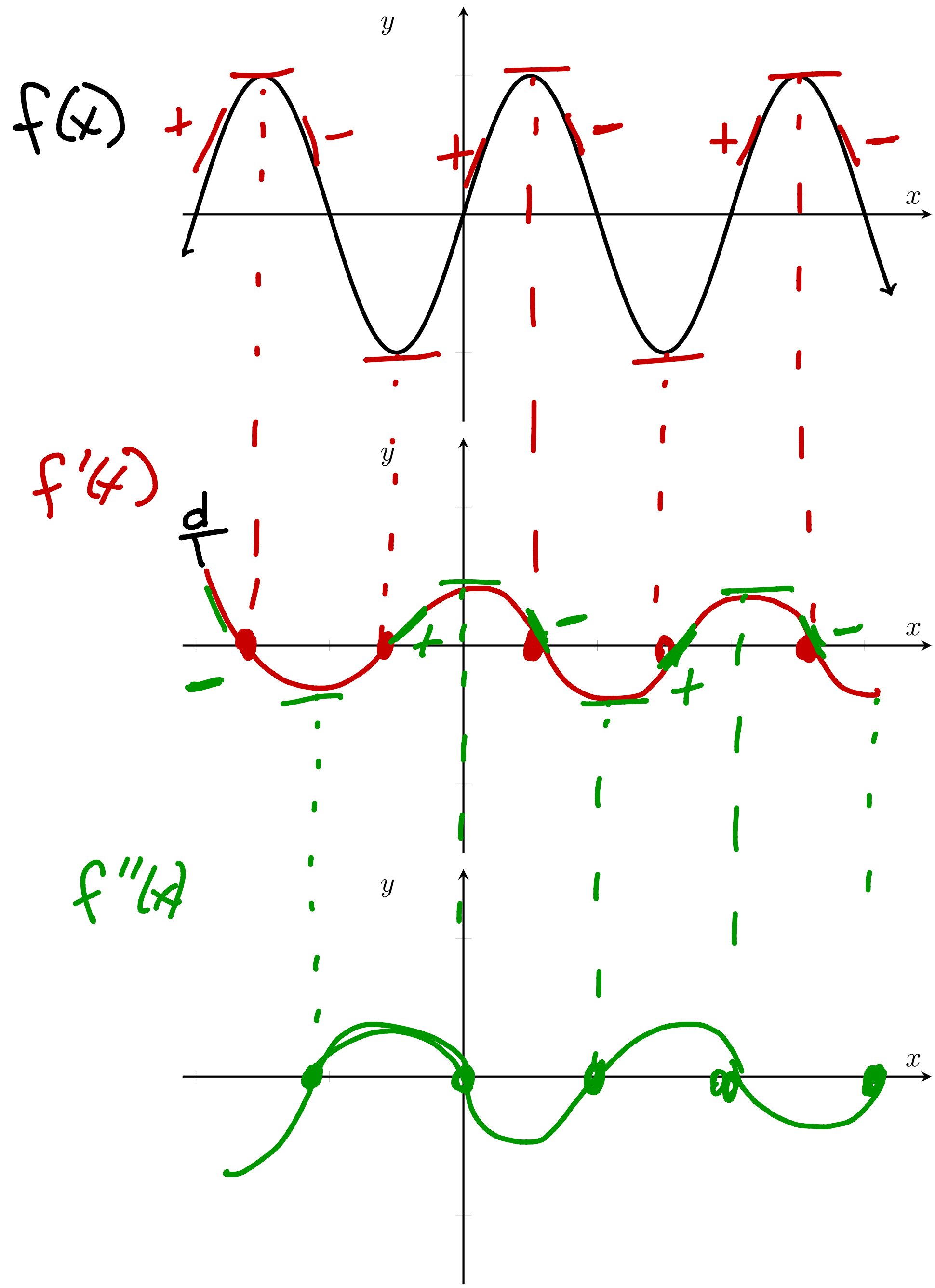
1. Find the derivative of $f(x) = \frac{1}{3}x^3 - \frac{x}{3} + \frac{\pi^2}{3}$. (What's wrong with the answer below?)

   answer: $f(x) = \frac{1}{3}x^3 - \frac{x}{3} + \frac{\pi^2}{3} = \frac{1}{3}\left(x^3 - x + \pi^2\right) = \frac{1}{3}(3x^2 - 1) = f'(x)$

   THIS equal sign is WRONG

   $f' \neq f$!!

2. (Good review for Midterm) The graph of $f(x)$ is sketched below. Graph its derivative $f'(x)$. Then, use your graph of $f'(x)$ to graph the derivative of $f''(x)$.



$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$

$\frac{d}{dx}\left[\cos(x)\right] = -\sin(x)$

$cx\left[\tan(x)\right] = \sec^2(x)$

$\frac{d}{dx}\left[\sec(x)\right] = (\sec(x))(\tan(x))$

$\frac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)$

$\frac{d}{dx}\left[\csc(x)\right] = -\cot(x)\csc(x)$

3. Find the derivative.

(a) $y = x^2 + 5\sin(x)$

$$y' = 2x + 5\cos(x)$$

(b) $f(\theta) = \theta\cos(\theta)$

$$f'(x) = 1 \cdot \cos(\theta) + \theta(-\sin(\theta))$$
$$= \cos(\theta) - \theta\sin(\theta)$$

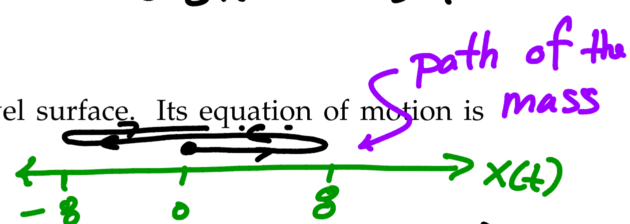
(c) $g(x) = \frac{\sin(x)}{x+1}$

$$g'(x) = \frac{(x+1)(\cos(x)) - (\sin(x))(1)}{(x+1)^2} = \frac{(x+1)\cos(x) - \sin(x)}{(x+1)^2}$$

(d) $H(x) = \frac{\sin(x)}{\cos(x)}$

$$H'(x) = \frac{\cos(x)(\cos(x)) - (\sin(x))(-\sin(x))}{(\cos(x))^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

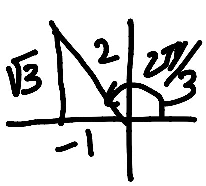
4. A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 8\sin(t)$, where t is in seconds and x is in centimeters.



(a) Find the velocity and acceleration at time t.

$$x(t) = 8\sin(t)$$
$$x'(t) = v(t) = 8\cos(t)$$
$$x''(t) = a(t) = -8\sin(t)$$

(b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$. In what direction is it moving at this time? Is it speeding up or slowing down?



$$x\left(\tfrac{2\pi}{3}\right) = 8\sin(2\pi/3) = 4\sqrt{3} \text{ cm}$$
$$v = x'(2\pi/3) = 8\cos(2\pi/3) = -4 \text{ cm/s}$$
$$a = x''\left(\tfrac{2\pi}{3}\right) = -8\sin\left(\tfrac{2\pi}{3}\right) = -8\left(\tfrac{\sqrt{3}}{2}\right) = -4\sqrt{3} \text{ cm/s/s}$$

ans:
- $v < 0$. So mass is moving left (negative x-direction)
- v and a have same sign. So mass is speeding up

More Derivatives

- $f(x) = x^2\tan(x) + \pi$

  $f'(x) = 2x\tan(x) + x^2 \cdot \sec^2 x + 0$

  $= 2x\tan(x) + x^2\sec^2 x$

- $y = \sin x$

  $y' = \cos(x) = y^{(5)} = y^{(9)} = \ldots$

  $y'' = -\sin(x)$

  $y''' = -\cos(x)$

  $y^{(4)} = \sin(x) = y^{(8)} = y^{(12)} = \ldots$

- $f(x) = \dfrac{x}{\sec(x)} = x\cos(x)$

rewrite w/ cosine + prod. rule

$f'(x) = 1 \cdot \cos x - x\sin x$

quotient rule

$f'(x) = \dfrac{1 \cdot \sec(x) - x\sec(x)\tan(x)}{\sec^2(x)}$

$= \dfrac{1 - x\tan(x)}{\sec(x)}$

are these the same?

$= \dfrac{1 - x\tan(x)}{\frac{1}{\cos x}} = \cos(x)\left[1 - x \cdot \dfrac{\sin(x)}{\cos(x)}\right]$

$= \cos(x) - x\sin(x)$

Yes. They are the same!