

SECTION 3-6: THE CHAIN RULE

1. Recall Two Versions of the Chain Rule

(Leibniz:) $y = f(u), u = g(x)$ (Newton:) $h(x) = f(g(x))$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

mini-examples: $h(x) = \tan(2x^3)$; $h(x) = (x + \sin(x))^5$

$y = \cos(u), u = 8x+1$

*finish probs from Fri.

2. Understanding what the "formulas" in the book are trying to communicate:

• If $h(x) = [g(x)]^n$, then $h'(x) = n[g(x)]^{n-1} \cdot g'(x)$.

• If $h(x) = \sin(g(x))$, then $h'(x) = \cos(g(x)) \cdot g'(x)$.

• $\frac{d}{dx} [\cos(u)] = -\sin(u) \cdot \frac{du}{dx}$

• $K(x) = h(f(g(x)))$, $K'(x) = h'(f(g(x))) \cdot f'(g(x)) \cdot g'(x)$.

3. $h(x) = \frac{2x(2x+1)^5}{\cos(2x+1)}$

* goal: Demonstrate a method for managing complication.

$$h'(x) = \frac{\cos(2x+1) \cdot \frac{d}{dx} [2x(2x+1)^5] - 2x(2x+1)^5 \cdot \frac{d}{dx} [\cos(2x+1)]}{\cos^2(2x+1)}$$

] quotient rule w/ placeholders

$$= \frac{\cos(2x+1) [2(2x+1)^5 + 2x \cdot \frac{d}{dx} [(2x+1)^5]] - 2x(2x+1)^5 (-\sin(2x+1) \cdot 2)}{\cos^2(2x+1)}$$

] prod. rule w/ placeholder

$$= \frac{\cos(2x+1) (2(2x+1)^5 + 2x(5(2x+1)^4(2)) + 4x(2x+1)^5 \sin(2x+1))}{\cos^2(2x+1)}$$

4. Find all x -values where the tangent to $f(x) = (x^2 - 4)^3$ is horizontal.

$$f'(x) = 3(x^2 - 4)^2(2x) = 0$$

$$x = 0, \pm 2$$

5. Use the table below to evaluate the derivatives of the given functions at the indicated value.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	2	-2	0	1
0	1	2	3	4
1	-1	-2	1	-4
2	0	4	3	5

(a) $h(x) = f(g(x) - 2x)$ at $a = 2$.

$$h'(x) = f'(g(x) - 2x)(g'(x) - 2)$$

$$h'(2) = f'(g(2) - 4) \cdot (g'(2) - 2)$$

$$= f'(3 - 4)(5 - 2) = f'(-1) \cdot 3 = -2 \cdot 3 = \boxed{-6}$$

(b) $k(x) = \left(\frac{f(x)}{g(x)}\right)^2$ at $a = 1$

$$= \frac{(f(x))^2}{(g(x))^2} = (f(x))^2 \cdot (g(x))^{-2}$$

$$k'(x) = 2 \cdot f(x) \cdot f'(x) \cdot (g(x))^{-2} + (f(x))^2 \cdot (-2) \cdot (g(x))^{-3} \cdot g'(x)$$

$$k'(1) = 2 \cdot f(1) \cdot f'(1) \cdot (g(1))^{-2} + (f(1))^2 \cdot (-2) \cdot (g(1))^{-3} \cdot g'(1)$$

$$= 2 \cdot (-1) \cdot (-2) \cdot (1)^{-2} + (-1)^2 \cdot (-2) \cdot (1)^{-3} \cdot (-4) = 4 + 8 = \boxed{12}$$