1. Goal: Understand and use the rule below:

If $f^{-1}(x)$ is the inverse of $f(x)$, then

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

Another way to express this is:
If you want to know the derivative of the inverse $((f^{-1})')$ at x, you could
i) find $f^{-1}(x) = y$ (the output of x)
ii) find $f'(y)$ (the derivative of f at the output value y)
iii) take the reciprocal: $\boxed{\frac{1}{f'(y)}}$. ← that's what you were looking for...

2. Fill out the rows of the chart below. Start with asterisked rows.

(a) $f(x) = x^3$

	$f(x)$	$f'(x)$	a-value	$b = f(a)$	$f'(a)$	point: (a, b)	slope at (a, b)
*	$f(x) = x^3$	$f'(x) = 3x^2$	2	$2^3 = 8$	$3 \cdot 2^2 = 12$	$(2, 8)$	12
	$f^{-1}(x)$	$(f')^{-1}(x)$	b-value	$a = f^{-1}(b)$	$f'^{-1}(b)$	point: (b, a)	slope at (b, a)
	$f^{-1}(x) = x^{1/3}$	$\frac{1}{3}x^{-2/3}$	8	$f^{-1}(8) = 2$	$\frac{1}{12}$	$(8, 2)$	$\frac{1}{12}$

$(f^{-1})'$

look!

(b) $f(x) = \frac{1}{x^2}$

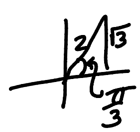
	$f(x)$	$f'(x)$	a-value	$b = f(a)$	$f'(a)$	point: (a, b)	slope at (a, b)
*	$f(x) = \frac{1}{x^2} = x^{-2}$	$f'(x) = -2x^{-3}$	3	$\frac{1}{3^2} = \frac{1}{9}$	$f'(3) = -\frac{2}{3^3} = -\frac{2}{27}$	$(3, \frac{1}{9})$	$-2/27$
	$f^{-1}(x)$	$(f')^{-1}(x)$	b-value	$a = f(b)$	$f'(b)$	point: (b, a)	slope at (b, a)
	$f^{-1}(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$	$-\frac{1}{2}x^{-3/2}$	$\frac{1}{9}$	$f^{-1}(1/9)$ $\frac{1}{\sqrt{1/9}} = 3$	$-\frac{1}{2}\left(\frac{1}{9}\right)^{-3/2} = -\frac{1}{2} \cdot \frac{27}{1} = -\frac{27}{2}$	$(\frac{1}{9}, 3)$	$-\frac{27}{2}$

look!

(c) $f(x) = \sin(x)$

	$f(x)$	$f'(x)$	a-value	$b = f(a)$	$f'(a)$	point: (a, b)	slope at (a, b)
*	$f(x) = \sin(x)$	$f'(x) = \cos(x)$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$f'(\pi/3) = \frac{1}{2}$	$(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$	$\frac{1}{2}$
	$f^{-1}(x)$	$(f')^{-1}(x)$	b-value	$a = f(b)$	$f'(b)$	point: (b, a)	slope at (b, a)
	$f^{-1}(x) = \arcsin(x) = \sin^{-1}(x)$	$\boxed{}$	$\sqrt{3}/2$	$\arcsin(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$		$(\frac{\sqrt{3}}{2}, \frac{\pi}{3})$	2

?



you can get here without knowing

Where does the formula

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$$

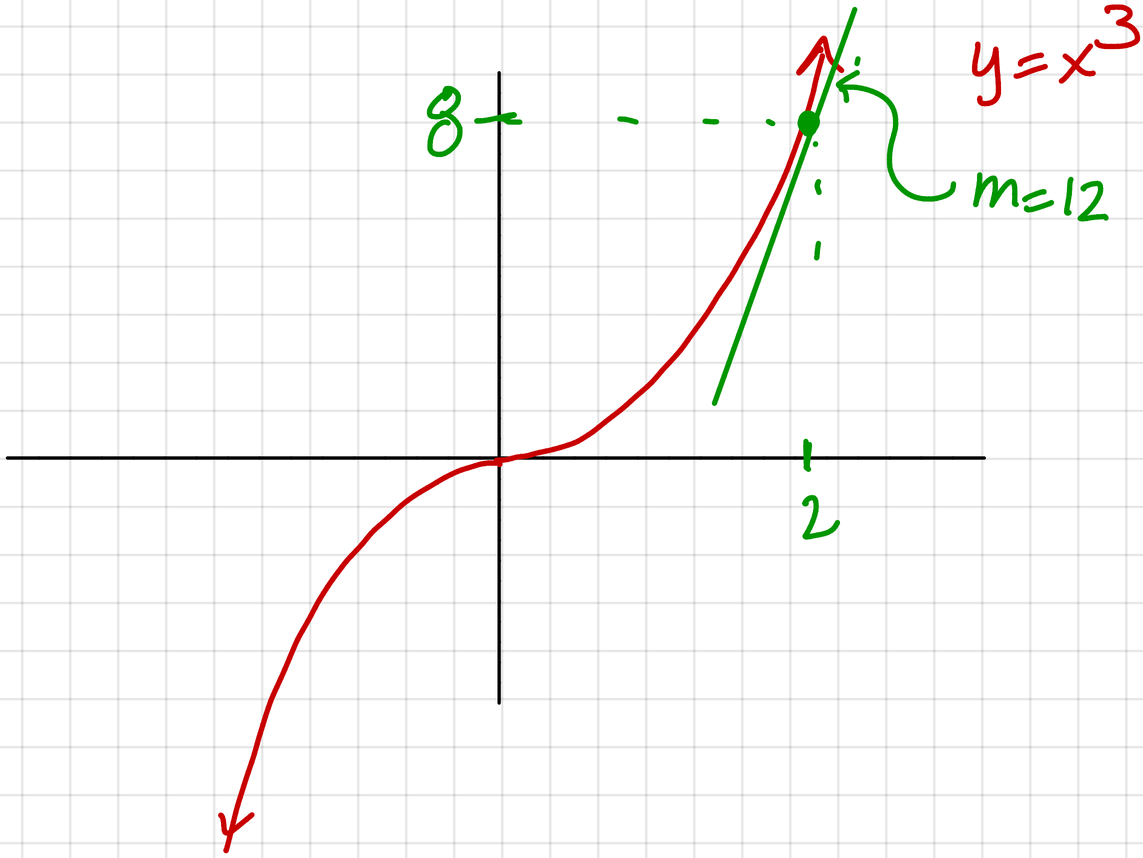
come from?

Answer: $f(f^{-1}(x)) = x$ property of inverses

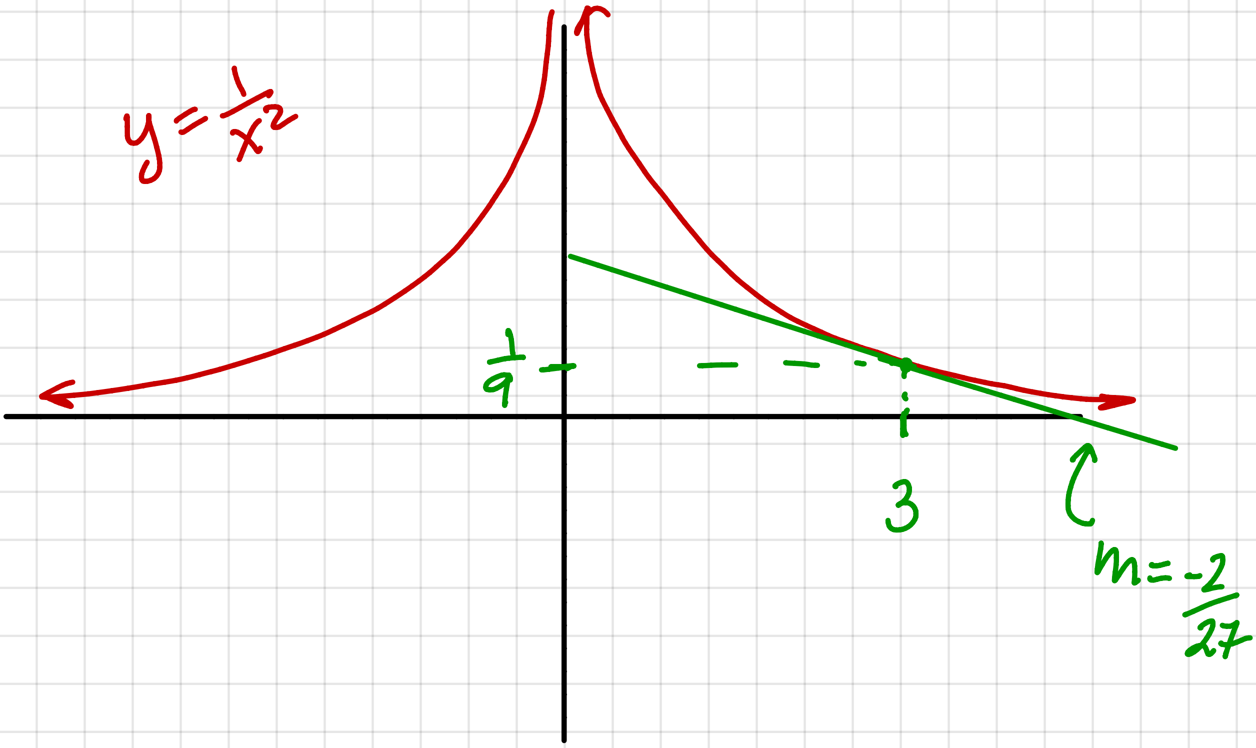
So $\underbrace{f'(f^{-1}(x)) \cdot (f^{-1})'(x)}_{\text{an application of the chain rule}} = 1$

Solve: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

Ⓐ

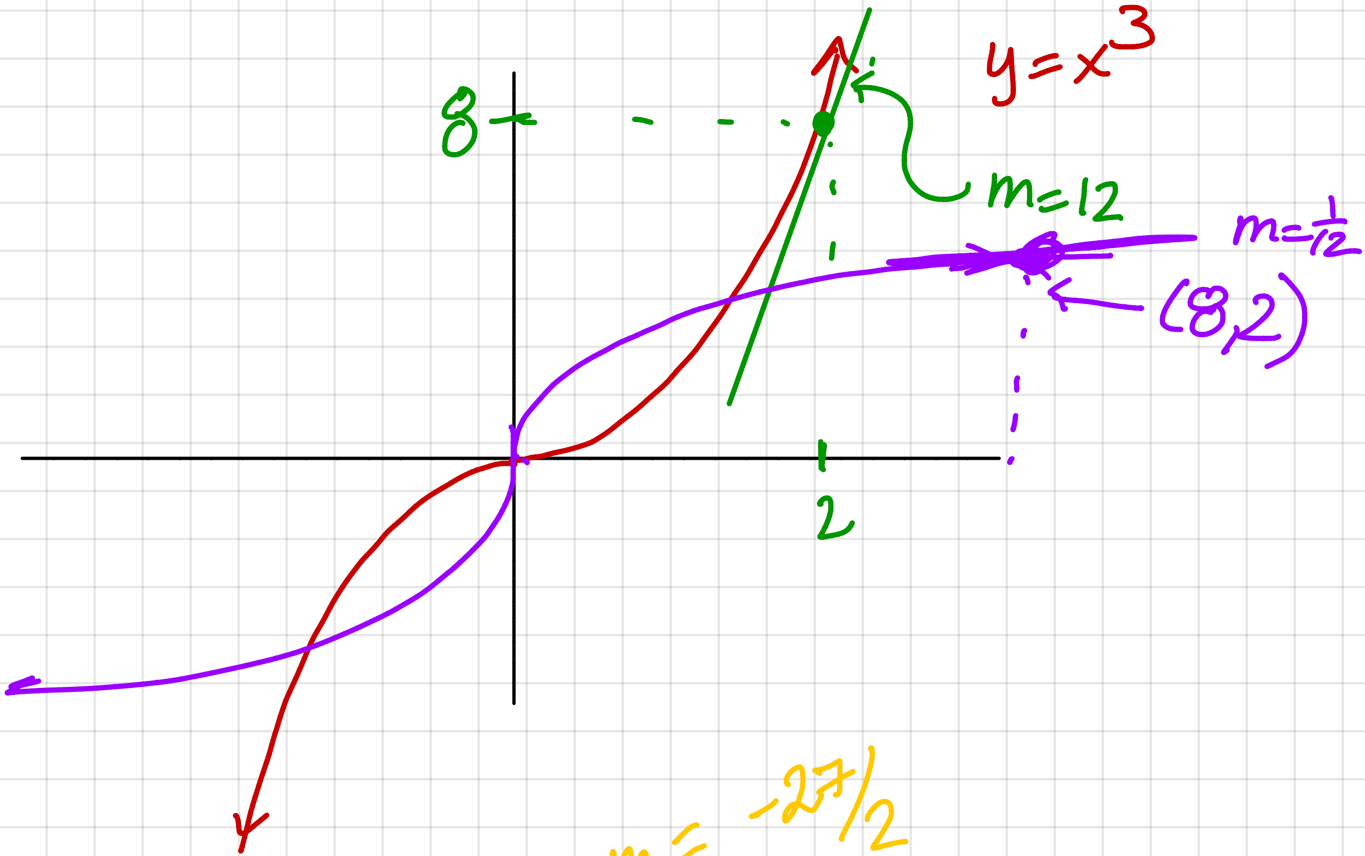

$y=x^3$
8
$m=12$
2

Ⓑ

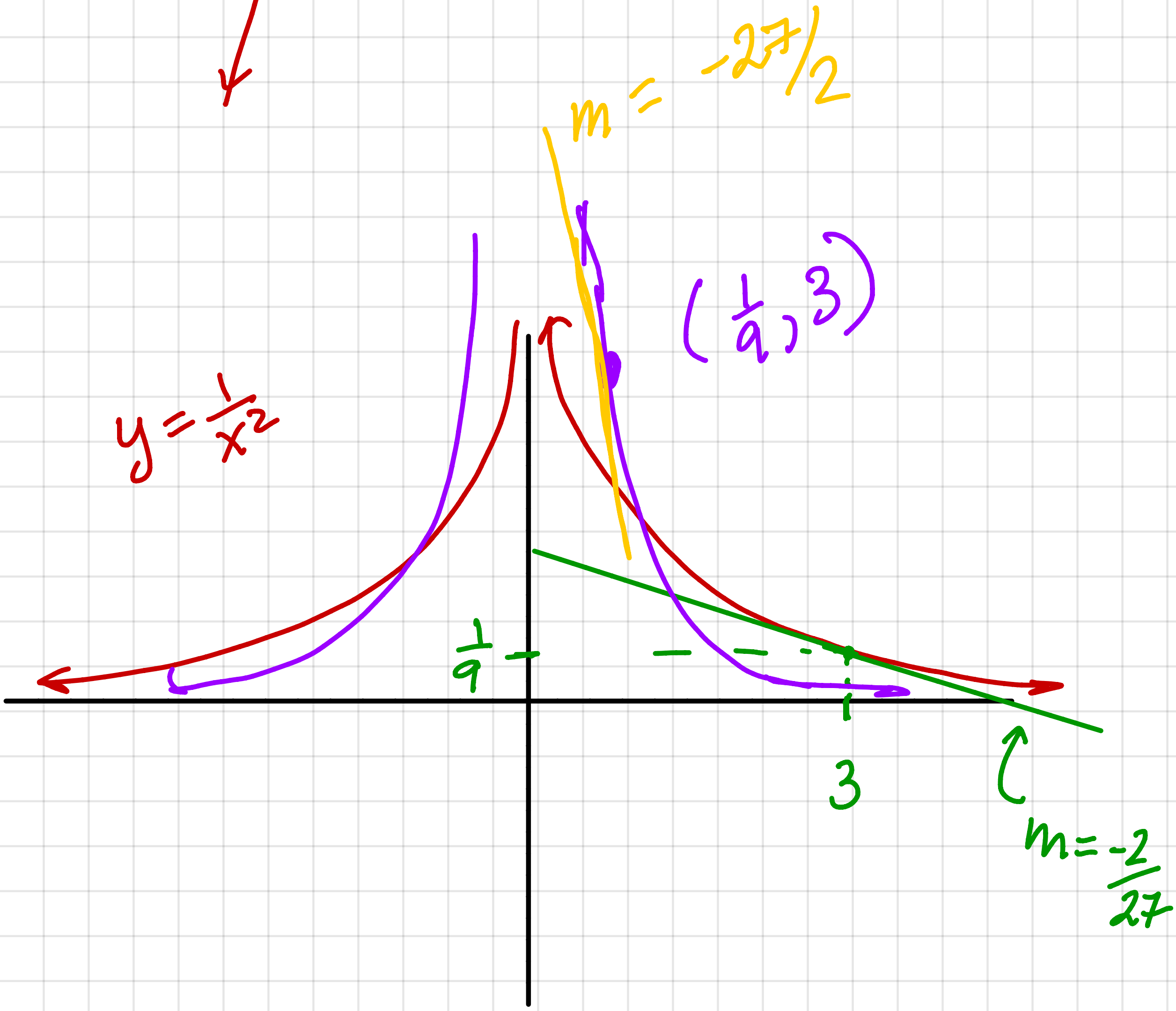

$y=\frac{1}{x^2}$
$\frac{1}{9}$
3
$m=\frac{-2}{27}$

Now, add the inverse functions in purple.

(a)

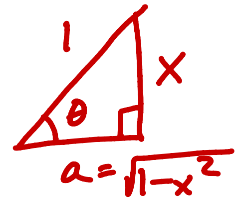


(b)



$x^2 + a^2 = 1^2$

$a = \sqrt{1-x^2}$



3. Use the rule from (1) to find a formula for the derivative of $g(x) = \sin^{-1}(x)$.

$f(x) = \sin(x)$, $f'(x) = \cos(x)$

$f^{-1}(x) = \sin^{-1}(x)$

$$f'(f^{-1}(x)) = \cos(\sin^{-1}(x)) = \sqrt{1-x^2} \quad // \quad (f^{-1})'(x) = \frac{1}{\sqrt{1-x^2}}$$

4. Rules for the arccosine and arctangent functions.

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}$$

* check that this fits w/ page 1.

5. Find the derivatives for each function below.

(a) $f(x) = \cos^{-1}(\sqrt{x})$

$$f'(x) = \frac{-1}{\sqrt{1-x}} \cdot \frac{1}{2} x^{-1/2} = \frac{-1}{\sqrt{x}\sqrt{1-x}}$$

(b) $f(x) = \left(\tan^{-1}(x)\right)^2$

$$f'(x) = 2\tan^{-1}(x) \cdot \frac{1}{1+x^2} = \frac{2\tan^{-1}(x)}{1+x^2}$$

(c) $f(x) = x\sin^{-1}(x)$

$$f'(x) = \sin^{-1}(x) + x \cdot \frac{1}{\sqrt{1-x^2}} = \sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}}$$

(d) $f(x) = \tan^{-1}(\frac{1}{x})$

$$f'(x) = \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \left(-x^{-2}\right) = \frac{-1}{x^2\left(1+\frac{1}{x^2}\right)}$$