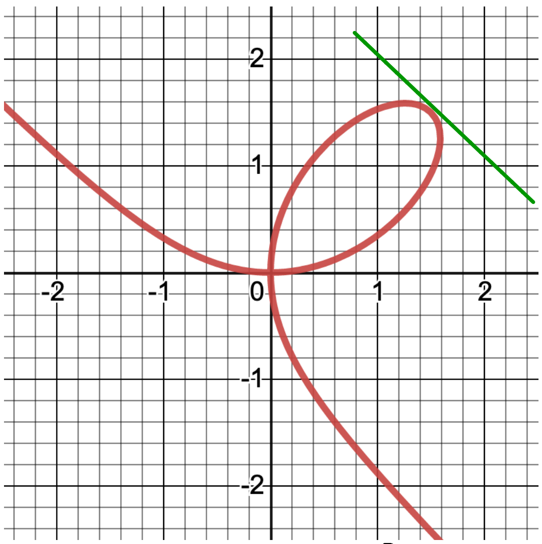
1. Motivating questions: How can we find slope of the tangent / velocity for a graph that looks like the one below?



Tangent line to $y^3 + x^3 = 3xy$ at $(3/2, 3/2)$?

• Find $\frac{dy}{dx}$.

$$3y^2\frac{dy}{dx} + 3x^2 = 3y + 3x\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 3x^2$$

$$(3y^2 - 3x)\frac{dy}{dx} = 3y - 3x^2$$

$$\frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}$$

$$\left.\frac{dy}{dx}\right|_{(\frac{3}{2},\frac{3}{2})} = \frac{3(\frac{3}{2}) - 3(\frac{3}{2})^2}{3(\frac{3}{2})^2 - 3(\frac{3}{2})} = -1$$

line: $y - \frac{3}{2} = -1(x - \frac{3}{2})$ or $y = -x + 3$

2. What is the derivative of: $(f(x))^3$?

$$3(f(x))^2 \cdot f'(x)$$ ← chain rule!

3. Repeat question 2 above but with Leibniz notation. What is dy/dx for: $(y)^3$?

$$3y^2 \cdot \frac{dy}{dx}$$

4. What is the derivative of $3xg(x)$?

$$3 \cdot 1 \cdot g(x) + 3x \cdot g'(x)$$ ← product rule

5. Repeat question 4 above but with Leibniz notation. What is dy/dx for: $3xy$?

$$3(1) \cdot y + 3x \cdot \frac{dy}{dx}$$

6. Find dy/dx for each expression below.

(a) $y\cos(x) + 2x = (y+1)^2$

$$\frac{dy}{dx}\cdot\cos(x) - y\sin(x) + 2 = 2(y+1)\cdot\frac{dy}{dx}$$

$$-y\sin x + 2 = 2(y+1)\frac{dy}{dx} - \cos(x)\frac{dy}{dx} = (2y+2-\cos(x))\frac{dy}{dx}$$ divide

So $\boxed{\dfrac{dy}{dx} = \dfrac{2-y\sin x}{2y+2-\cos(x)}}$

(b) $x + \tan(xy) = 5$

$$1 + \sec^2(x)\left[1\cdot y + x\cdot\frac{dy}{dx}\right] = 0$$ distribute

$$1 + y\sec^2(x) + x\sec^2(x)\frac{dy}{dx} = 0$$

$$1 + y\sec^2(x) = -x\sec^2(x)\frac{dy}{dx}$$

$\boxed{\dfrac{dy}{dx} = \dfrac{1+y\sec^2(x)}{-x\sec^2(x)}}$

7. For the equation $x^2 + xy + y^2 = 9$,

(a) find the x intercept(s)

when $y=0$. So $x^2 = 9$ or $x = \pm 3$

(b) Find the slope of the tangent lines at the x-intercepts.

Find dy/dx.

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$(x+2y)\frac{dy}{dx} = -2x-y\text{; So } \frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

at $(\pm 3, 0)$ $\frac{dy}{dx} = -2$

(c) Write the equations of the tangent lines at the x-intercepts.

$y = -2(x+3)$, $y = -2(x-3)$

(d) Sketch a picture of the curve and its tangent lines from part (c)