1. Quick Review of Implicit Differentiation: Find dy/dx for $x2 - y^3 = x\sin(y)$.

$$2x - 3y^2\frac{dy}{dx} = 1\cdot\sin(y) + x\cos(y)\frac{dy}{dx}$$

$$2x - \sin(y) = \left[3y^2 + x\cos(y)\right] dy/dx \longrightarrow \frac{dy}{dx} = \frac{2x - \sin(y)}{3y^2 + x\cos(y)}$$

2. Derivative Rules for Exponential Functions

$$\frac{d}{dx}\left[e^x\right] = e^x \longleftarrow f(x) = e^x$$ has the property that for every x-value its associated y-value is also the slope of the tangent. (ie $f(x) = f'(x)$)

$$\frac{d}{dx}\left[a^x\right] = \ln(a)\, a^x \longleftarrow$$ For $f(x) = a^x$, relationship is proportional (ie. by $\ln(a)$)

Compare $a > 1$ and $a < 1$.

$$\frac{d}{dx}\left[e^{g(x)}\right] = g'(x)\, e^{g(x)}$$

The Chain Rule.

$$\frac{d}{dx}\left[a^{g(x)}\right] = g'(x)\ln(a)\, a^{g(x)}$$

3. Examples:

(a) $y = x^4 e^x$ $f \cdot g$

$f' \cdot g + f \cdot g'$

$$y' = 4x^3 \cdot e^x + x^4 \cdot e^x = e^x(4x^3 + x^4)$$

(b) $y = e^{x^2}$

$$y' = e^{x^2} \cdot 2x$$

(c) $y = 5^{-x}$

$$y' = (\ln 5)\, 5^{-x}(-1) = (-\ln 5)\, 5^{-x}$$

(d) $f(x) = x^5 + 5^x$

$$f'(x) = 5x^4 + (\ln 5)\, 5^x$$

Think about the difference

$y = x^3$	$y = e^{2x}$
$y' = 3x^2$	$y' = 2e^{2x}$
$y'' = 6x$	$y'' = 4e^{2x}$
$y''' = 6$	$y''' = 8e^{2x}$
$y^{(4)} = 0 = y^{(5)}$	$y^{(k)} = 2^k e^{2x}$

4. Let $P(t) = P_0 e^{kt}$. Write $P'(t)$ in terms of $P(t)$.

$$\underbrace{P'(t)}_{\text{the rate of change of } P} = (P_0 e^{kt})(k) = k \cdot P(t)$$

is   proportional to   the existing population

5. Write $y = \log_2(x)$ and $y = \ln(x)$ in terms of exponential functions.

$\boxed{y = \log_2 x} \longleftrightarrow \boxed{2^y = x}$ $\boxed{y = \ln(x)} \longleftrightarrow \boxed{x = e^y}$

6. Use the expressions in #5 to find formulas for the derivatives of $y = \log_2(x)$ and $y = \ln(x)$.

$2^y = x$

$\ln(2) \cdot 2^y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\ln(2) \cdot 2^y} = \frac{1}{\ln(2)\, x}$

$x = e^y$

$1 = e^y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$

$\frac{d}{dx}\left[\log_2 x\right] = \frac{1}{(\ln(2))x}$; $\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}$

$\frac{d}{dx}\left[\log_2(g(x))\right] = \frac{g'(x)}{\ln(2)\, g(x)}$

$\frac{d}{dx}\left[\ln(g(x))\right] = \frac{g'(x)}{g(x)}$

7. Examples:

(a) $y = x\ln(x)$ $y' = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$

(b) $y = \log(x^2 - 5)$ $= \log_{10}(x^2 - 5)$

$y' = \frac{2x}{\ln(10)\,(x^2-5)}$

(c) $y = \ln\left(\frac{x(x^2+1)^3}{100(x+1)}\right)$ $= \ln(x) + 3\ln(x^2+1) - \ln(100) - \ln(x+1)$

$y' = \frac{1}{x} + \frac{2x}{x^2+1} - 0 - \frac{1}{x+1} = \frac{1}{x} + \frac{2x}{x^2+1} - \frac{1}{x+1}$

(d) $y = (\sin(x))^x$

$\ln y = x \ln(\sin(x))$

$\frac{1}{y} \cdot \frac{dy}{dx} = 1 \cdot \ln(\sin(x)) + x \cdot \frac{\cos(x)}{\sin(x)}$

$\frac{dy}{dx} = y\left(\ln(\sin(x)) + x\cot(x)\right)$

$\frac{dy}{dx} = (\sin(x))^x\left[\ln(\sin x) + x\cot(x)\right]$