A strategy.

- Draw a $\boxed{\text{picture}}$.
- Identify what you $\boxed{\text{want}}$ and what you $\boxed{\text{know}}$
- $\boxed{\text{Take derivative with respect to } t.}$
- $\boxed{\text{Solve}}$ for what you $\boxed{\text{want}}$.

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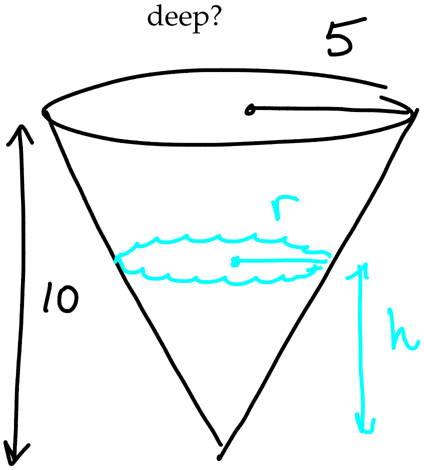
1. Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



want $\frac{dh}{dt}$ when $h = 6$ ft.

know $\frac{dV}{dt} = 9 \text{ ft}^3/\text{min}$

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$$

Use similar Δ's

$$\frac{r}{h} = \frac{5}{10} \quad \text{or} \quad r = \frac{1}{2}h$$

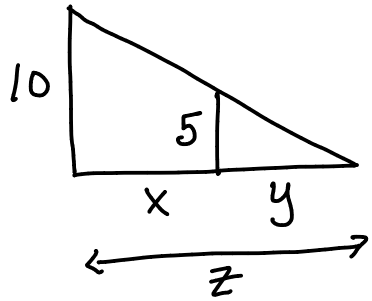
So $V = \frac{\pi}{12} h^3$

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \boxed{\frac{dh}{dt}}$$

$$9 = \frac{\pi}{4}(6)^2 \cdot \frac{dh}{dt}$$

$$\text{or} \quad \frac{dh}{dt} = \frac{9 \cdot 4}{36\pi} = \frac{1}{\pi} \text{ ft/min}$$

2. A street light is mounted at the top of a 10-ft-tall pole. A woman 5 ft tall walks away from the pole along a straight path at a speed of 5 ft/s. How fast is the tip of her shadow moving when she is 40 ft from the pole?



want $\dfrac{dz}{dt}$ when $z=40$

Know $\dfrac{dx}{dt} = 5\text{ft/s}$

$$y = z - x$$

$$\frac{z-x}{5} = \frac{z}{10}$$

or

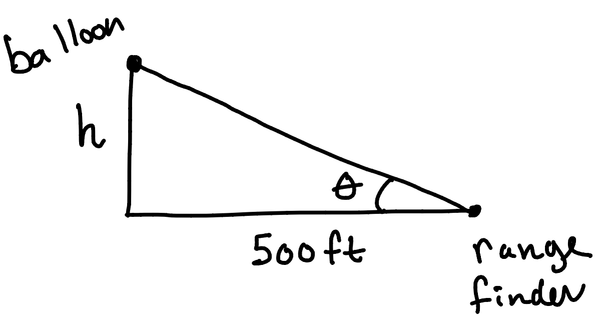
$$10z - 10x = 5z$$

or $5z = 10x$ or $z = 2x$

$$\frac{dz}{dt} = 2 \cdot \frac{dx}{dt}$$

Plug in: $\dfrac{dz}{dt} = 2 \cdot 5 = 10 \text{ ft/s}$

3. A hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 radians/min. How fast is the balloon rising at that moment?



Want $\dfrac{dh}{dt}$ when $\theta = \pi/4$

and $\dfrac{d\theta}{dt} = 0.14$ rad/min.

$$\tan\theta = \frac{h}{500}$$

$$(\sec^2\theta) \cdot \frac{d\theta}{dt} = \frac{1}{500} \boxed{\frac{dh}{dt}}$$

$$\frac{dh}{dt} = 500 \sec^2\theta \frac{d\theta}{dt} = 500 \sec^2\left(\frac{\pi}{4}\right)(0.14) = (500)(0.14)(\sqrt{2})^2$$

$$= 140 \text{ ft/min}$$

$$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos(\pi/4)}$$

$$= \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$