1. The Second Derivative Test → Suppose $f'(a)=0$, f'' is continuous in interval containing a

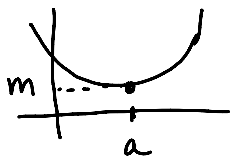
then:

(i) $f(a)$ is a local min if $f''(a)>0$

(ii) $f(a)$ is a local max if $f''(a)<0$

(iii) If $f''(a)=0$, the test is inconclusive; We don't know if $f(a)$ is a local max, local min, or neither.

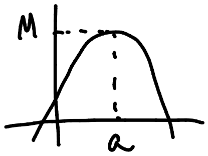
motivated by picture:



local min at $x=a$

concave up

$f''>0$



local max at $x=a$

concave down

$f''<0$

2. Use the Second Derivative Test to find the local extrema for $f(x) = -3x^5 + 5x^3$.

$f'(x) = -15x^4+15x^2 = -15x^2(x^2-1) = -15x^2(x+1)(x-1)$

$f''(x) = -60x^3+30x = -30x(2x^2-1)$

C.pts: $x=0,+1,-1$; $f''(0)=0$, $f''(1)<0$, $f''(-1)>0$

Conclusion of 2nd Der Test

$f(1)$ is a local max

$f(-1)$ is a local min

at $x=0$, the test tells us nothing. (In fact, $f(0)$ is neither max nor min.)

3. For the function $f(x) = \sqrt[3]{x}(1-x)$ determine (a) intervals where f is increasing/decreasing, (b) the locations of any local extrema (c) intervals where f is concave up / concave down (d) inflection points. Then use technology to confirm your answers.

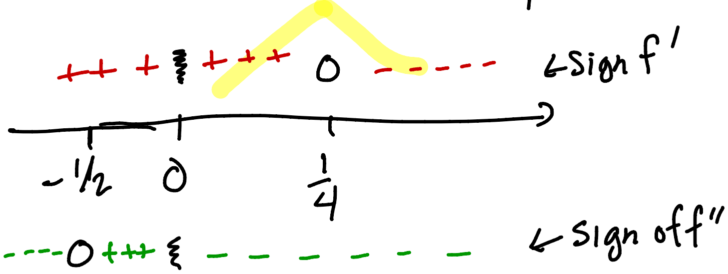
$f(x) = x^{1/3}(1-x) = x^{1/3} - x^{4/3}$

$f'(x) = \frac{1}{3}x^{-2/3} - \frac{4}{3}x^{1/3} = \frac{1}{3x^{2/3}} - \frac{4}{3}x^{1/3} = \frac{1-4x}{3x^{2/3}}$ ← So this determines sign. ← always +

$f''(x) = -\frac{2}{9}x^{-5/3} - \frac{4}{9}x^{-2/3} = -\left(\frac{2}{9x^{5/3}} + \frac{4}{9x^{2/3}}\right) = -\frac{2}{9}\left(\frac{1}{x^{5/3}} + \frac{2}{x^{2/3}}\cdot\frac{x}{x}\right) = -\frac{2}{9}\left(\frac{1+2x}{x^{5/3}}\right)$

(1) C.pts: f' undefined at $x=0$.

$f'=0$ when $x=\frac{1}{4}$



(2) f'' undefined @ $x=0$

$f''=0$ when $x=-\frac{1}{2}$

Answers:

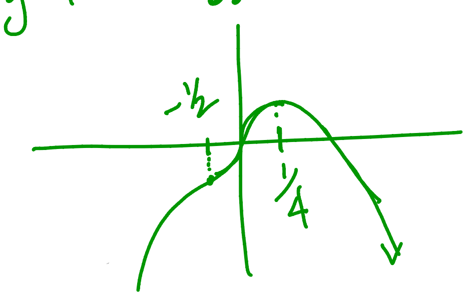
(a) f is ↑ on $(-\infty,\frac{1}{4})$, ↓ on $(\frac{1}{4},\infty)$

(b) f has a local max @ $x=\frac{1}{4}$

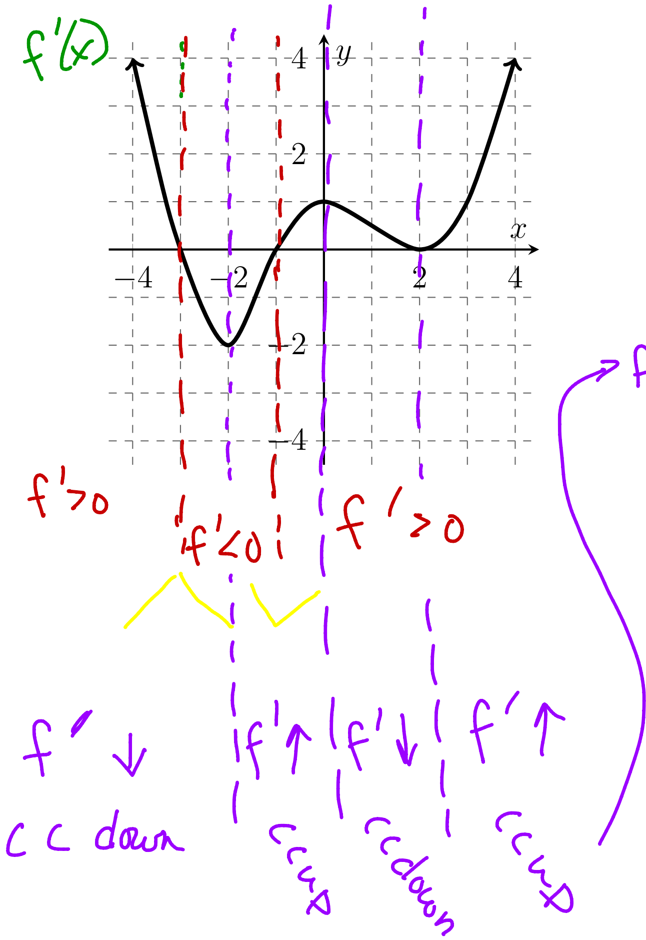
(c) f is cc up on $(-\frac{1}{2},0)$, cc down on $(-\infty,-\frac{1}{2})\cup(0,\infty)$

(d) f has inflection points at $x=-\frac{1}{2}$ and $x=0$.

graph (exaggerated a bit.)



4. Below is the graph of the *derivative* of f, $f'(x)$. Use this graph to answer the questions.



(a) On what intervals is $f(x)$ increasing? decreasing?

f ↑ on $(-\infty,-3)\cup(-1,\infty)$, ↓ on $(-3,-1)$

(b) Determine the location of local extrema of f.

f has local min at $x=-1$, local max at $x=-3$

(c) On what intervals is $f(x)$ concave up? concave down?

f cc up on $(-2,0)\cup(2,\infty)$, cc down $(-\infty,-2)\cup(0,2)$

(d) Determine the location of any inflection points of f.

f has inflection pts at

$x=-2, 0, 2$

5. Sketch a graph that satisfies *all* of the properties below.

(a) $f(2) = f(4) = 0$
(b) $f'(x) > 0$ if $x < 3$
(c) $f'(3)$ does not exist
(d) $f'(x) < 0$ if $x > 3$
(e) $f''(x) > 0$ for $x \neq 3$.