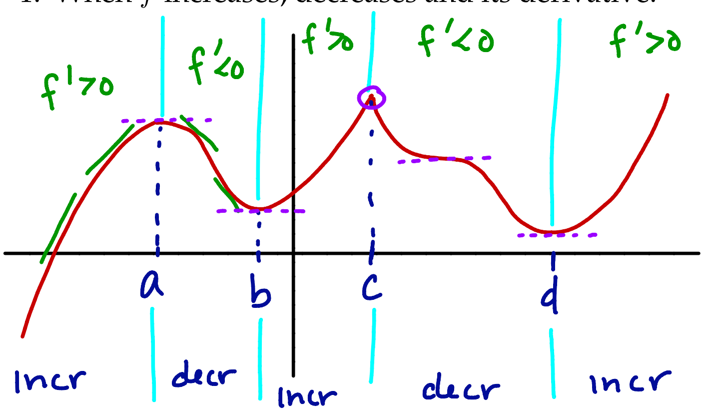
1. When f increases, decreases and its derivative.



• Where is f increasing? decreasing?

• Where is f'>0? f'<0?

• Where is f'=0 or undefined?

*Observe the local max's & min's occur when f' changes sign.

2. The First Derivative Test

Let f be continuous on interval I with crit. pt. x=c.

• If f'>0 on left of c and f'<0 on right (+ to −), then f(c) local max.

• If f'<0 on left of c and f'>0 on right (− to +), then f(c) local min.

• If f' doesn't change sign, f has no extremum at x=c.

3. For the function $f(x) = \frac{2}{3}x^3 + x^2 - 12x + 7$:

(a) Determine the intervals where $f(x)$ is increasing or decreasing.

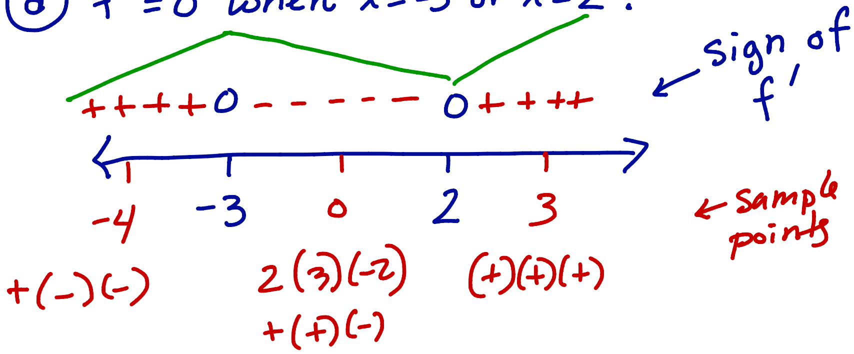
(b) Use the First Derivative Test to identify the location of all local extrema.

(c) Use technology to confirm your work.

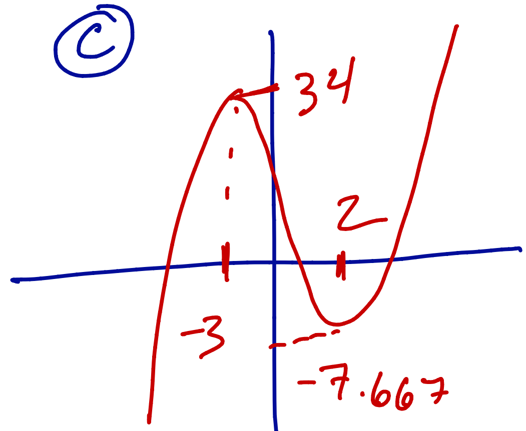
Algorithm • find c.pts
• check sign of f'
• draw conclusion

$f'(x) = 2x^2 + 2x - 12 = 2(x^2 + x - 6) = 2(x+3)(x-2)$

ⓐ $f' = 0$ when $x = -3$ or $x = 2$.



ⓒ



$$\boxed{\begin{array}{c} f(x) \text{ is increasing on } (-\infty,-3) \cup (2,\infty) \\ \text{decreasing on } (-3,2) \end{array}}$$

ⓑ How I think about it in green.
f has a local max at x = −3
a local min at x = 2.

This is an application of First Derivative Test.

4. Identify all local extrema for $f(x) = x^2 e^{-x}$

$f'(x) = 2xe^{-x} + x^2(-1)e^{-x}$

$\quad = xe^{-x}(2-x)$

$f'(x) = 0$ when $x = 0$ or $x = 2$.

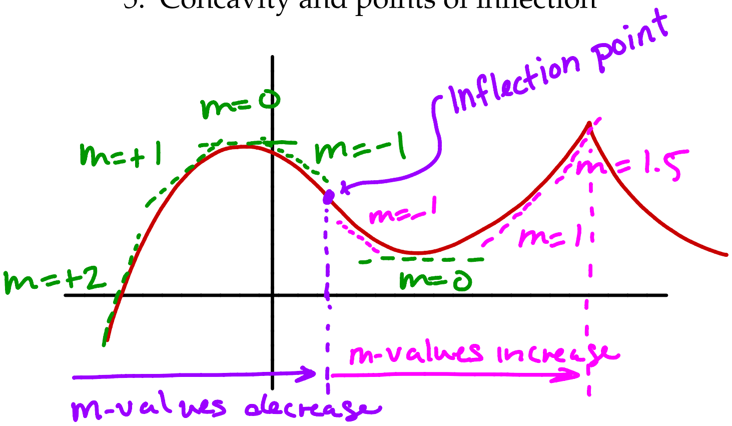
$- - - -\ 0\ + + + +\ 0\ - - - - -$ ← sign of $f'(x)$

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

$(-)(+)(+) \quad (+)(+)(+) \quad (+)(+)(-)$

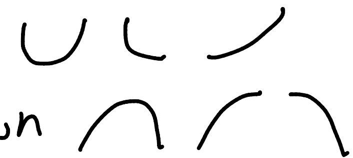
$\boxed{f \text{ has a local min at } x=0, \text{ local max at } x=2}$

5. Concavity and points of inflection



concave up

concave down



- Observe that when f is concave up, f' is increasing; so $f'' > 0$.
- When f is concave down, f' is decreasing; so $f'' < 0$.
- Inflection points ≡ where concavity changes

6. Test for Concavity

f is twice differentiable on interval I

i. If $f'' > 0$ on I, then f is concave up on I.

ii. If $f'' < 0$ on I, then f is concave down on I.

7. Determine the intervals for which the function $f(x) = \frac{2}{3}x^3 + x^2 - 12x + 7$ is concave up and concave down. Identify the x-coordinate of any inflection points.

$f''(x) = 4x + 2 = 2(2x+1)$

$f'' = 0$ when $x = -\frac{1}{2}$

$- - - -\ 0\ + + + +$ ← sign of f''

$-1 \quad -\frac{1}{2} \quad 0$

f is ccup on $(-\frac{1}{2}, \infty)$, ccdown on $(-\infty, -\frac{1}{2})$

Inflection point at $x = -\frac{1}{2}$

8. Do the same for $f(x) = x^2 e^{-x}$.

Use $f'(x) = e^{-x}(2x - x^2)$

So $f''(x) = -e^{-x}(2x - x^2) + e^{-x}(2 - 2x)$

$\quad = e^{-x}(x^2 - 4x + 2) = e^{-x}(x - (2+\sqrt{2}))(x - (2-\sqrt{2}))$

$f''(x) = 0 \quad x = 2 \pm \sqrt{2}$

$+ + +\ 0\ - - -\ 0\ + + +$

$2-\sqrt{2} \quad 2+\sqrt{2}$

f ccup on $(-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$

ccdown on $(2-\sqrt{2}, 2+\sqrt{2})$

Infl. pts at $x = 2 \pm \sqrt{2}$