1. Three Principles (a is a constant)

   - If a is a constant, then $\lim\limits_{x\to\pm\infty} ax = \pm\infty$
   - $\lim\limits_{x\to\pm\infty} \frac{1}{x} = 0$
   - If $\lim\limits_{x\to\pm\infty} f(x) = a$ and $\lim\limits_{x\to\pm\infty} g(x) = \pm\infty$, then $\lim\limits_{x\to\pm\infty} \frac{f(x)}{g(x)} = 0$

So $\lim\limits_{x\to\pm\infty} ax^2 = \pm\infty$ and

$\therefore \lim\limits_{x\to\pm\infty} \frac{1}{x^2} = 0$

2. Use the Principles above to evaluate the limits below.

   (a) $\lim\limits_{x\to\infty} \frac{-x}{3x-5x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim\limits_{x\to\infty} \frac{\overset{\to 0}{-\frac{1}{x}}}{\underset{\downarrow 0}{\frac{3}{x}} - \underset{\downarrow -5}{5}} = \frac{0}{-5} = 0$

   (b) $\lim\limits_{x\to\infty} \frac{2x^2-x}{3x-5x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim\limits_{x\to\infty} \frac{2-\overset{\to 0}{\frac{1}{x}}}{\underset{\downarrow 0}{\frac{3}{x}} - 5} = \frac{2}{-5} = \frac{-2}{5}$

   (c) $\lim\limits_{x\to\infty} \frac{2x^3-x}{3x-5x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim\limits_{x\to\infty} \frac{\overset{\to\infty}{2x} - \overset{\to 0}{\frac{1}{x}}}{\underset{\downarrow 0}{\frac{3}{x}} \underset{\to -5}{-5}} = \infty$

   (d) $\lim\limits_{x\to\infty} \frac{3x+\sin(x)}{x} = \lim\limits_{x\to\infty} \underset{\downarrow 3}{3} + \underset{\downarrow 0}{\frac{\sin(x)}{x}} = 3$

   (e) $\lim\limits_{x\to-\infty} \frac{2x+1}{\sqrt{x^2+1}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to-\infty} \frac{2+\overset{\to 0}{\frac{1}{x}}}{-\sqrt{1+\underset{\downarrow 0}{\frac{1}{x^2}}}} = -2$

   (f) $\lim\limits_{x\to\infty} \frac{2e^x+1}{1-3e^x} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}} = \lim\limits_{x\to\infty} \frac{2+\overset{\to 0}{\frac{1}{e^x}}}{\underset{\to 0}{\frac{1}{e^x}} - 3} = \frac{2}{-3} = \frac{-2}{3}$

* Rational Functions behave in predictable ways

$\lim\limits_{x\to\pm\infty} \frac{\text{polynomial}}{\text{polynomial}}$ ← divide by highest degree in denominator.

3. Limits at Infinity and Horizontal Asymptotes: If $\lim_{x\to\infty} f(x) = L$, then $y = L$ is a horizontal asymptote for $f(x)$.

$x \to -\infty$

4. Find all asymptotes of $f(x) = \frac{x}{3-x}$ and *justify* your answers.

V.a. : $x=3$

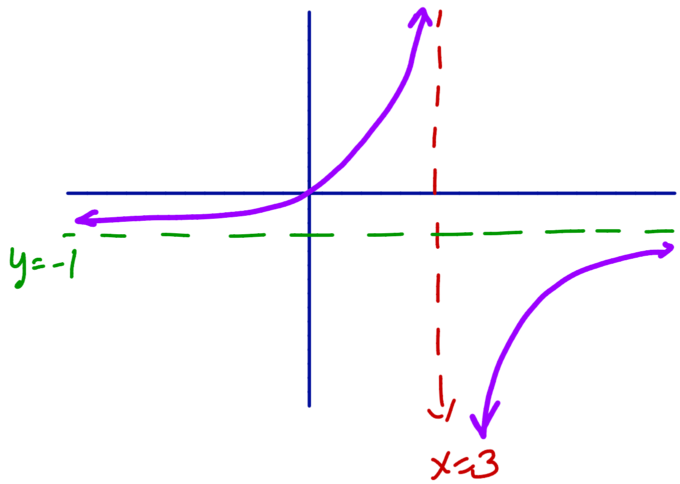
justification:

$$\lim_{x\to 3^+} \frac{x}{3-x} = -\infty$$

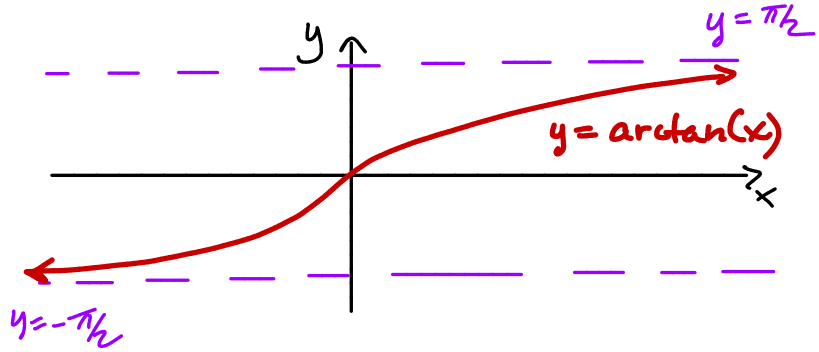
h.a.: $y=-1$

justification:

$$\lim_{x\to\infty} \frac{x}{3-x} = -1$$



5. Find $\lim_{x\to-\infty} \tan^{-1}(x)$ $= -\pi/2$



6. Given $f(x) = \frac{2x+1}{x^2+6x+5}$, $f'(x) = \frac{-2(x^2+x-2)}{(x^2+6x+5)^2}$, $f''(x) = \frac{2(2x^3+3x^2-12x-29)}{(x^2+6x+5)^3}$. (Hint: $f''(x) = 0$ when $x = 2.7034$.) Identify important features of $f(x)$ like: asymptotes, local extrema, inflection points, and make a rough sketch.

asymptotes:

h.a: $y=0$

$x^2+6x+5=(x+5)(x+1)$

v.a. $x=-1, x=-5$

concavity:

$- - - \ \} + + + + + \ \} - - - - \ 0 + + +$ ← Sign f''

$-5 \qquad -1 \qquad 2.7034$

inflection point at $x=2.7034$

(not at $x=-5$ or -1 b/c not in domain)

extrema:

$x^2+x-2=(x+2)(x-1)$

$f'=0$ when $x=-2, x=1$

$- - - \ 0 + + + \ 0 - - -$ ← Sign f'

$-2 \qquad 1$

local min @ $x=-2$

local max @ $x=1$