

SECTION 4.6: LIMITS AT INFINITY AND ASYMPTOTES (DAY 1)

1. Three Principles (a is a constant)

- If a is a constant, then $\lim_{x \rightarrow \pm\infty} ax =$
- $\lim_{x \rightarrow \pm\infty} \frac{1}{x} =$
- If $\lim_{x \rightarrow \pm\infty} f(x) = a$ and $\lim_{x \rightarrow \pm\infty} g(x) = \pm\infty$, then $\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)} =$

2. Use the Principles above to evaluate the limits below.

(a)
$$\lim_{x \rightarrow \infty} \frac{-x}{3x - 5x^2}$$

(b)
$$\lim_{x \rightarrow \infty} \frac{2x^2 - x}{3x - 5x^2}$$

(c)
$$\lim_{x \rightarrow \infty} \frac{2x^3 - x}{3x - 5x^2}$$

(d)
$$\lim_{x \rightarrow \infty} \frac{3x + \sin(x)}{x}$$

(e)
$$\lim_{x \rightarrow -\infty} \frac{2x + 1}{\sqrt{x^2 + 1}}$$

(f)
$$\lim_{x \rightarrow \infty} \frac{2e^x + 1}{1 - 3e^x}$$

3. Limits at Infinity and Horizontal Asymptotes: If $\lim_{x \rightarrow \infty} f(x) = L$, then

4. Find all asymptotes of $f(x) = \frac{x}{3-x}$ and *justify* your answers.

5. Find $\lim_{x \rightarrow -\infty} \tan^{-1}(x)$

6. Given $f(x) = \frac{2x+1}{x^2+6x+5}$, $f'(x) = \frac{-2(x^2+x-2)}{(x^2+6x+5)^2}$, $f''(x) = \frac{2(2x^3+3x^2-12x-29)}{(x^2+6x+5)^3}$. (Hint: $f''(x) = 0$ when $x = 2.7034\dots$ Identify important features of $f(x)$ like: asymptotes, local extrema, inflection points, and make a rough sketch.