SECTION 4.7 OPTIMIZATION (DAY 1)

1. A Framework for Approaching Optimization
   (a) Identify the quantity to be minimized or maximized.
   (b) Chose notation and explain what it means.
   (c) Write the thing you want to maximize or minimize **as a function of one variable**, including a reasonable **domain**.
   (d) Use calculus to answer the question and *justify* that your answer is correct.

   Read the problem two or three times. Draw pictures. Label them. Pick specific numerical examples, to make the problem concrete. Be creative. Try more than just one approach. Organization matters.

2. Find two positive numbers whose sum is 110 and whose product is a maximum.

x, y positive numbers

$x + y = 110$, $y = 110 - x$

maximize $P = xy$

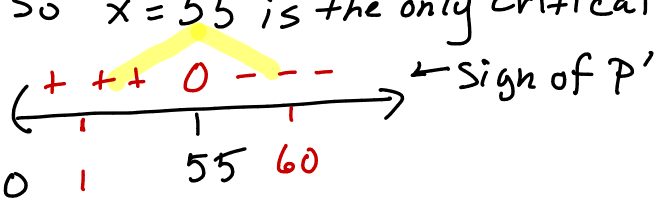
$P(x) = x(110 - x)$ domain $(0, \infty)$

$= 110x - x^2$

$P'(x) = 110 - 2x$

$P' = 0$ when $110 - 2x = 0$

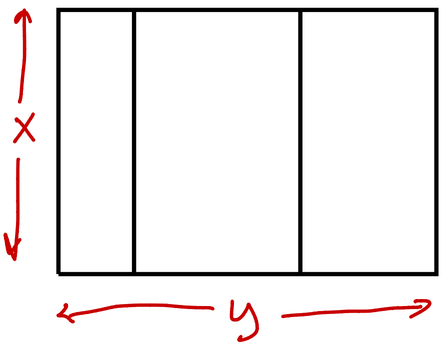
So $x = 55$ is the only critical point.



So P has a local max at $x = 55$ by the first derivative test. Since P has only one crit. pt on $(0, \infty)$, the local max is an absolute maximum.

ANS: The two numbers whose sum is 110 and whose product is a maximum are $x = 55$ and $y = 55$.

3. A rancher has 800 feet of fencing with which to enclose three adjacent rectangular corrals. See figure below. What dimensions should be used so that the enclosed area will be a maximum?



$800 = 2y + 4x$, $y = 400 - 2x$

maximize area $A = xy$

$A(x) = x(400-2x) = 400x - 2x^2$ $\quad D: [0, 200]$

$A'(x) = 400 - 4x$

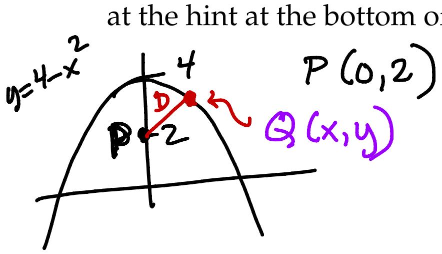
$A' = 0$ when $x = 100$. So $x = 100$ is the only crit. pt of A on $[0, 200]$.

Because $A(x)$ is a parabola that opens down, $A(x)$ must have an absolute max at $x = 100$.

Answer: The dimensions the maximize area are $x = 100$ ft and $y = 200$ ft.

$4 - \frac{3}{2} = 8$

4. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$? (Get started on this problem and once you have a function – that is, you have made it through part (d) of the Framework – look at the hint at the bottom of the page.)



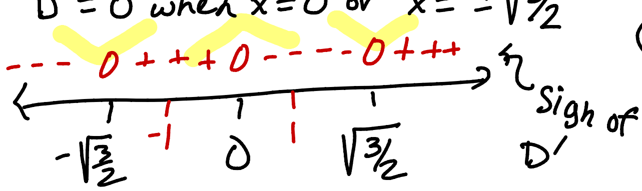
minimize the distance from P to Q. (Follow hint!)

$D = (x-0)^2 + (y-2)^2 = x^2 + (y-2)^2$ $\quad$ (distance)^2

$D(x) = x^2 + (4 - x^2 - 2)^2 = x^2 + (2 - x^2)^2$ $\quad D: (-\infty, \infty)$

$D'(x) = 2x + 2(2-x^2)(-2x) = -6x + 4x^3 = \underline{2x(2x^2 - 3)}$

$D' = 0$ when $x = 0$ or $x = \pm\sqrt{3/2}$



local min at $x = \pm\sqrt{3/2}$

note: $x = 0$ is a local max!

Since D is decreasing on $(-\infty, -\sqrt{3/2})$ and increasing on $(\sqrt{3/2}, \infty)$, D has absolute min at $x = \pm\sqrt{3/2}$.

ANSWER: The points $\left(\pm\sqrt{\frac{3}{2}}, \frac{5}{2}\right)$ are the points on $y = 4 - x^2$ closest to P(0,2).

HINT: Whenever you are asked to maximize or minimize distance, it is nearly ALWAYS easier to maximize or minimize the *square* of the distance. Why?