Section 4.8 L'Hôpital's Rule (Day 1)

1. Give two functions $f(x)$ and $g(x)$ with the property that when you try to evaluate the limit as $x \to 1$ *by direct substitution* you get $0/0$ but that, in fact, $\lim_{x\to 1} f(x) = 2$ and $\lim_{x\to 1} g(x) = -14$.

$$\lim_{x\to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1} x+1 = 2$$

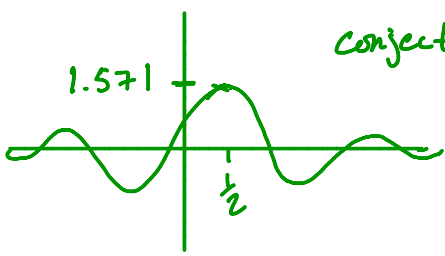
$$\lim_{x\to 1} \frac{(x-15)(x-1)}{x-1} = \lim_{x\to 1} x-15 = -14$$

lesson: When you get $\frac{0}{0}$ by substitution, you cannot conclude the limit is zero, or ∞, or 2 or any particular number. Compare this to $\lim_{x\to\infty} \frac{1}{x}$, or $\lim_{x\to\infty} \frac{5}{x^3}$

2. Try to evaluate the following limits below by substitution. Use technology to draw the graphs and make a conjecture about what you think the limit should be.

*(a) $\displaystyle\lim_{x\to 1/2} \frac{\cos(\pi x)}{1-2x} = \frac{\cos \pi/2}{1-1} = \frac{0}{0}$

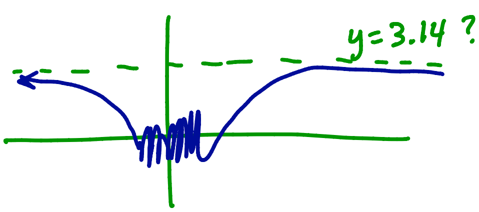
*Note The factor/cancel method won't work here.



conjecture: $\displaystyle\lim_{x\to\frac{1}{2}} \frac{\cos(\pi x)}{1-2x} = 1.571$

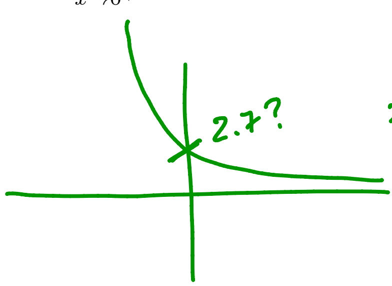
* (b) $\displaystyle\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right) = \infty \cdot \sin(0) = \infty \cdot 0$

* No idea what algebra to do here!



Conjecture: $\displaystyle\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right) \approx 3.14$ (maybe π?)

* (c) $\displaystyle\lim_{x\to 0^+} (1+x)^{1/x} = (1)^{+\infty}$



$\displaystyle\lim_{x\to 0^+} (1+x)^{1/x} \approx 2.6$

3. L'Hôpital's Rule says:

We want to evaluate

$$\underbrace{\lim_{x\to a} \frac{f(x)}{g(x)}}_{\text{form } \frac{0}{0}.} \quad \text{where } \lim_{x\to a} f(x)=0 \text{ and } \lim_{x\to a} g(x)=0.$$

Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \underbrace{\lim_{x\to a} \frac{f'(x)}{g'(x)}}_{\text{*not* the quotient rule.}}$$

provide this limit is a number or $\pm\infty$.

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$$\lim_{x\to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1} \frac{x^2-1}{x-1} \overset{H}{=} \lim_{x\to 1} \frac{2x}{1} = 2$$

$$\lim_{x\to 1} \frac{(x-15)(x-1)}{x-1} = \lim_{x\to 1} \frac{x^2-16x+15}{x-1} \overset{H}{=} \lim_{x\to 1} \frac{2x-16}{1} = -14$$

$$\lim_{x\to 1/2} \frac{\cos(\pi x)}{1-2x} \overset{H}{=} \lim_{x\to \frac{1}{2}} \frac{-\pi \sin(\pi x)}{-2} = \frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\approx 1.5708$$

Do not apply L'Hop willy-nilly

$$\lim_{x\to \frac{1}{2}} \frac{\cos(\pi x)}{1+2x} \overset{H}{=} \cancel{\lim_{x\to \frac{1}{2}} \frac{-\pi \sin(\pi x)}{2} = -\frac{\pi}{2}} \approx -1.5708$$

not in form $\frac{0}{0}$

substitution

$$= \frac{\cos(\pi/2)}{2} = \frac{0}{2} = 0$$