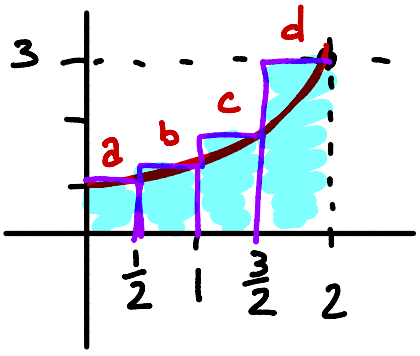
SECTION 5.1: APPROXIMATING AREAS

Using rectangles to estimate areas of curvy curves.

1. For all parts of this problem, the goal is to estimate the area below $f(x) = \frac{1}{2}x^2 + 1$ and above the x-axis on the interval $[0, 2]$.

   (a) (R_4) Use $n = 4$ rectangles and right-hand endpoints.



area = w · h
= $\frac{1}{2} \cdot f(x)$

area under curve $\approx$ area of rectangle a + area rectangle b + area rectangle c + area rectangle d

w · h + w · h + w · h + w · h

$= \frac{1}{2} \cdot f(\frac{1}{2}) + \frac{1}{2} \cdot f(1) + \frac{1}{2} f(\frac{3}{2}) + \frac{1}{2} f(2)$

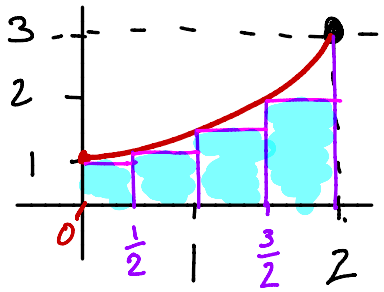
$= \frac{1}{2}\left[\left(\frac{1}{2}\left(\frac{1}{2}\right)^2+1\right) + \left(\frac{1}{2}\cdot 1^2+1\right) + \left(\frac{1}{2}\cdot\left(\frac{3}{2}\right)^2+1\right) + \left(\frac{1}{2}(2)^2+1\right)\right]$

(h_a, h_b, h_c, h_d)

$= \frac{1}{2}\left[\frac{9}{8} + \frac{3}{2} + \frac{17}{8} + 3\right] = 3.875 = R_4$

overestimate.

   (b) (L_4) Use $n = 4$ rectangles and left-hand endpoints.



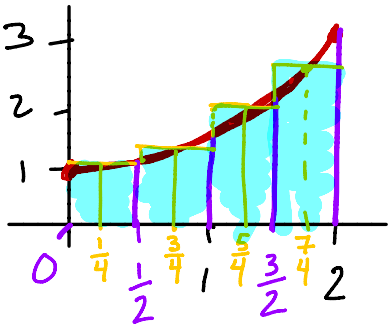
area under curve $\approx \frac{1}{2} f(0) + \frac{1}{2} f(\frac{1}{2}) + \frac{1}{2} f(1) + \frac{1}{2} f(\frac{3}{2})$

$= \frac{1}{2}\left[1 + \frac{9}{8} + \frac{3}{2} + \frac{17}{8}\right] = 2.875$

$= L_4$

underestimate!

   (c) (M_4) Use $n = 4$ rectangles and midpoints endpoints.

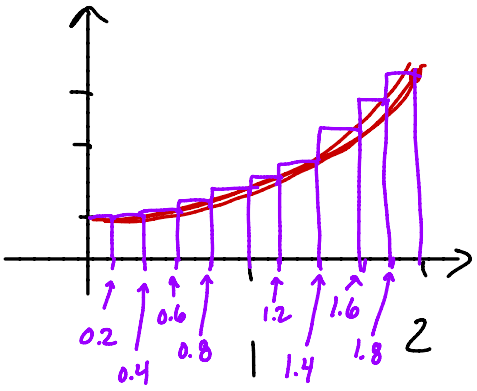


area under curve $\approx \frac{1}{2}\left[f(\frac{1}{4}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4})\right]$

$= \frac{1}{2}\left[\left(\frac{1}{2}\left(\frac{1}{4}\right)^2+1\right) + \left(\frac{1}{2}\left(\frac{3}{4}\right)^2+1\right) + \left(\frac{1}{2}\left(\frac{5}{4}\right)^2+1\right) + \left(\frac{1}{2}\left(\frac{7}{4}\right)^2+1\right)\right]$

$= 3.3125 = M_4$

(d) Use R_{10}



area under curve $\approx (0.2)[f(0.2)+f(0.4)+f(0.6)+f(0.8)+f(1)+f(1.2)+f(1.4)+f(1.6)+f(1.8)+f(2)]$

$\overset{\text{comp. tool}}{\uparrow} = (0.2)(17.7) = 3.54$

$$\sum_{i=1}^{10} w_i h_i$$

Summation Notation!

2. Oil leaked out of a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate atn 2-hour time intervals are shown in the table. Estimate how much oil leaked out. What method are you using? Is is an over estimate? Underestimate? Can you tell?

time, t, (in hours)	0	2	4	6	8	10
rate, $r(t)$, (in liters/hour)	8.7	7.6	6.8	6.2	5.7	5.3

There are so many choices.

- liters leaked $\approx R_5 = 2(7.6+6.8+6.2+5.7+5.3) = 63.2$ liters (underestimate.)

- liters leaked $\approx L_5 = 2[8.7+7.6+6.8+6.2+5.7+5.3] = 70$ liters

overestimate.