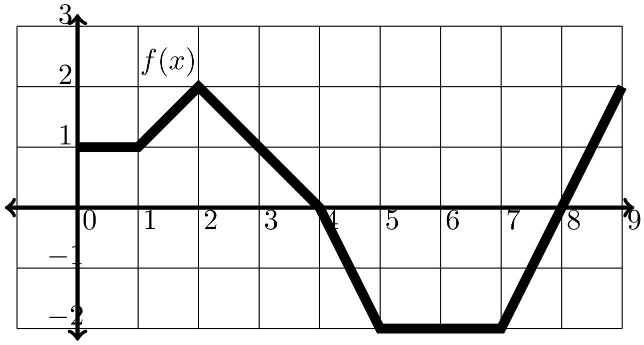
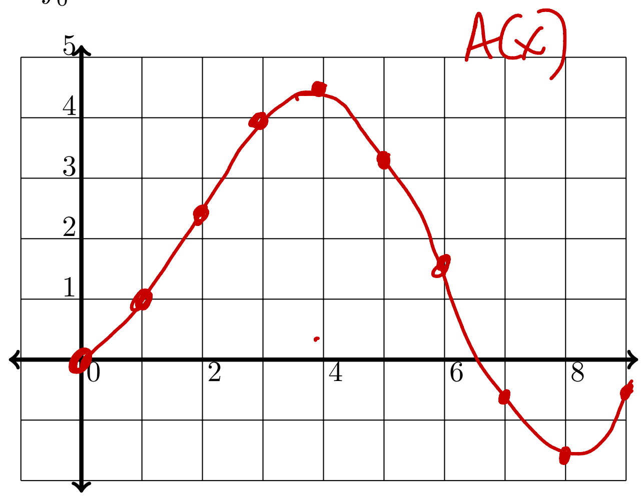
Section 5.3: The Fundamental Theorem of Calculus

1. Let $f(x)$ be given by the graph below and define $A(x) = \int_0^x f(t)dt$.





(a) Compute the following using the graph of $f(x)$. Then sketch $A(x)$.

$f(0) =$ 1	$f(5) =$ -2	$A(0) =$ 0	$A(5) =$ 4.5-1=3.5
$f(1) =$ 1	$f(6) =$ -2	$A(1) =$ 1	$A(6) =$ 3.5-2=1.5
$f(2) =$ 2	$f(7) =$ -2	$A(2) =$ 1+1.5=2.5	$A(7) =$ 1.5-2=-0.5
$f(3) =$ 1	$f(8) =$ 0	$A(3) =$ 4	$A(8) =$ -0.5-1=-1.5
$f(4) =$ 0	$f(9) =$ 2	$A(4) =$ 4.5	$A(9) =$ -0.5

(b) Where is $A(x)$ increasing? $(0,4) \cup (8,9)$

(c) Describe f when $A(x)$ is increasing. $f > 0$

(d) Where is $A(x)$ decreasing? $(4,8)$

(e) Describe f when $A(x)$ is decreasing. $f < 0$

(f) Where does $A(x)$ have a local maximum? $x=4$

(g) Describe f when $A(x)$ has a local max. f crosses x-axis from + to −

(h) Where does $A(x)$ have a local minimum? $x=8$

(i) Describe f when $A(x)$ has a local min. f crosses x-axis from − to +

(j) What can you say about the **rate of change** of $A(x)$? $A'(x)=f(x)$.

Here, t is considered a dummy variable.

2. The Fundamental Theorem of Calculus (part 1):

If $F(x) = \int_a^x f(t)\,dt$,

So $F(x)$ is the net signed area under $f(x)$ on $[a, x]$.

then $F'(x) = f(x)$.

(Does require $f(t)$ be continuous!)

Alternative formulation:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

3. Find the derivative of each function below.

(a) $g(x) = \int_2^x (t^2 - \tan(t))\,dt$

$g'(x) = x^2 - \tan(x)$

(b) $h(x) = \int_0^{\sin(x)} \sqrt{t^3+1}\,dt = \left(\sqrt{\sin^3 x + 1}\right)(\cos x)$

Chain rule.

extended explanation

let $u = \sin(x)$

$h(u) = \int_0^u \sqrt{t^3+1}\,dt.$

Then $\frac{dF}{dx} = \frac{dF}{du}\cdot\frac{du}{dx}$

$= \sqrt{u^3+1}\cdot\cos(x)$

$= \left(\sqrt{\sin^3 x+1}\right)(\cos x)$

4. The Fundamental Theorem of Calculus (part 2):

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a), \text{ where } F'(x) = f(x)$$

F is any antiderivative of $f(x)$

much easier than: $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)w_i$

(Does require f to be continuous)

5. Evaluate the integrals.

(a) $\int_0^\pi \sin(x)\,dx$

$= -\cos(x)\Big]_0^\pi$

$= -\cos(\pi) - (-\cos 0)$

$= -0 + 1 = 1$

(b) $\int_{-1}^3 x + e^x\,dx$

$= \frac{1}{2}x^2 + e^x\Big]_{-1}^3 = \left(\frac{1}{2}\cdot 3^2 + e^3\right) - \left(\frac{1}{2}(-1)^2 + e^{-1}\right)$

$= \frac{9}{2} - \frac{1}{2} + e^3 - \frac{1}{e}$

$= 4 + e^3 - \frac{1}{e}$

2. The Fundamental Theorem of Calculus (part 1):

3. Find the derivative of each function below.

(a) $g(x) = \int_2^x (t^2 - \tan(t))\, dt$

(b) $h(x) = \int_0^{\sin(x)} \sqrt{t^3 + 1}\, dt$

4. The Fundamental Theorem of Calculus (part 2):

5. Evaluate the integrals.

(a) $g(x) = \int_0^{\pi} \sin(x)\, dx$

(b) $h(x) = \int_{-1}^{3} x + e^x\, dx$