Section 5.4: The Net Change Theorem

1. Quick Review: Evaluate the following.

(a) $\int \left(\frac{x}{3} - \sin(x)\right)\, dx$

$= \frac{1}{6}x^2 + \cos(x) + C$

(b) $\int_0^5 (3 - e^x)\, dx$

$= 3x - e^x\Big]_0^5$

$= (15 - e^5) - (0 - e^0)$

$= 16 - e^5$

(c) $\frac{d}{dx}\left(\int_1^{x^2} (\ln(t))\, dt\right)$

$= \ln(x^2)(2x)$

$= 4x\ln(x)$

2. Assume $P'(t)$ gives the rate of change in a population of ants over time where time t is measured in days and $P'(t)$ is measured in hundreds of ants per day. Use the table below to answer the questions.

t	0	7	14	21	28	35
P'(t)	0	1.9	2.4	2.7	3.0	3.2

(a) Interpret $P'(14) = 2.4$. On the 14th day, the population of ants is increasing at a rate of 240 ants per day.

(b) Estimate how much the ant population increased in the first three weeks. Include units with your answer.

est #1: $7(0 + 1.9 + 2.4) = 30.1$ hundreds of ants $= 3010$ ants

est #2: $7(1.9 + 2.4 + 2.7) = 49$ hundreds of ants $= 4900$ ants

(c) What would $\int_0^{21} P'(t)\, dt$ represent? (There are many ways to answer this question. Think of as many as you can. Include units this that is appropriate)

$\int_0^{21} P'(t)dt = P(21) - P(0)$; area under $P'(t)$ on $[0,21]$;

The exact increase in the population of ants in the first 3 weeks given in hundreds of ants.

(d) What would $P(t)$ represent? What is $P(14)$?

$P(t)$ would be the population of ants at time t.

$P(14)$? We don't know... Did the colony start with 0 ants? 1000? No one told us. (ie $\int P'(t)\,dt = P(t) + C$??)

3. The Net Change Theorem:

$\boxed{\int_a^b F'(t)\,dt} = F(b) - F(a)$

or

$F(b) = F(a) + \boxed{\int_a^b F'(t)dt}$

$\rightarrow$ This will tells how much F changed from $t=a$ to $t=b$. (ie. Net Change). It does NOT tell us the value of $F(b)$. We need $F(a)$ to do this.

(or something else.)

4. Snow is falling on my garden at a rate of

$$A(t) = 10e^{-2t}$$

kilograms per hour for $0 \leq t \leq 2$, where t is measured in hours.

(a) Find $A(1)$ and interpret in the context of the problem.

$A(1) = 10e^{-2\cdot 1} = \frac{10}{e^2}$ kg/hr. At hour 1, snow is falling on my garden at a rate of $\frac{10}{e^2}$ kg/hr

(b) If $m(t)$ is the total mass of snow on my garden, how are $m(t)$ and $A(t)$ related to each other?

$m'(t) = A(t)$ or $m(t) = m(0) + \int_0^t A(s)\,ds$

(c) What does $m(2) - m(0)$ represent?

The mass of snow that accumulated on my garden in these two hours.

(d) Find an antiderivative of $A(t)$.

$\int 10e^{-2t}\,dt = -5e^{-2t}$

(e) Compute the total amount of snow accumulation from $t = 0$ to $t = 1$.

$\int_0^1 10e^{-2t}\,dt = -5e^{-2t}\Big]_0^1 = -5e^{-2} - (-5e^0) = 5\left(1 - \frac{1}{e^2}\right)$

(f) Compute the total amount of snow accumulation from $t = 0$ to $t = 2$.

$\int_0^2 10e^{-2t}\,dt = -5e^{-2t}\Big]_0^2 = -5e^{-4} + 5 = 5\left(1 - \frac{1}{e^4}\right)$

(g) From the information given so far, can you compute $m(2)$?

No. We need to know how much snow was already on the garden when snow started falling.

(h) Suppose $m(0) = 9$. Compute $m(1)$ and $m(2)$.

$m(1) = 9 + 5\left(1 - \frac{1}{e^2}\right)$

$m(2) = 9 + 5\left(1 - \frac{1}{e^4}\right)$

5. A airplane is descending. Its rate of change of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

(a) If $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

(b) What physical quantity does $\int_1^3 r(t)\, dt$ represent?

(c) Compute $A(3) - A(1)$.

(d) What is the height of the plane when $t = 3$?