SECTION 5.5: SUBSTITUTION (I.E. UNDOING THE CHAIN RULE)

1. (like #259 in hmwk)

   (a) Verify that the formula is correct: $\int \frac{2x}{\sqrt{x^2-1}}\,dx = 2\sqrt{x^2-1}+C$

   If $F(x)=2(x^2-1)^{1/2}+C$

   then $F'(x)=2\cdot\frac{1}{2}(x^2-1)^{-1/2}\cdot 2x = \frac{2x}{\sqrt{x^2-1}}$

   (b) Use the substitution $u = x^2-1$ to rewrite *the entire integral* in terms of u. Then integrate the integral with the new variables.

   $\int 2x(x^2-1)^{-1/2}\,dx = \int (x^2-1)^{-1/2}\,\underline{2x\,dx} = \int u^{-1/2}\,du = 2u^{1/2}+C$

   $= 2(x^2+1)^{1/2}+C$

   let $u=x^2-1$

   $du=\underline{2x\,dx}$

2. Explain why the formula is not correct: $\int \sqrt{x^2+1}\,dx = \frac{1}{3}(x^2+1)^{3/2}+C$

   If $F(x)=(x^2+1)^{3/2}$, $F'(x)=\frac{1}{3}\frac{3}{2}(x^2+1)^{1/2}(2x) = x\sqrt{x^2+1}+C$

3. Goals: (a) Practice u-substitution (b) Practice *sophisticated* u-substitution (c) Practice substitution with both indefinite and definite integrals (d) Develop intuition about how to choose u.

4. $\int t^3\cos(t^4+1)\,dt = \int \cos(\underline{\underline{t^4+1}})\,\underline{\underline{t^3dt}} = \int \cos(u)\cdot\frac{1}{4}\,du$

   $= \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin(u)+C$

   $= \frac{1}{4}\sin(t^4+1)+C$

   Let $u=\underline{\underline{t^4+1}}$

   $du=4t^3dt$

   $\frac{1}{4}du=\underline{\underline{t^3dt}}$

5. $\int \sin^2(x)\cos(x)\,dx = \int(\underline{\underline{\sin(x)}})^2\,\underline{\underline{\cos x\,dx}} = \int u^2\,du$

   $= \frac{1}{3}u^3+C = \frac{1}{3}(\sin x)^3+C$

   Let $u=\underline{\underline{\sin(x)}}$

   $du=\underline{\underline{\cos(x)\,dx}}$

6. $\displaystyle\int (x-1)(x^2-2x)^{10}\,dx = \frac{1}{2}\int u^{10}\,du = \frac{1}{22}u^{11}+C$

let $u = x^2-2x$

$du = (2x-2)\,dx$

$\frac{1}{2}\,du = (x-1)\,dx$

$= \frac{1}{22}(x^2-2x)^{11}+C$

7. $\displaystyle\int \frac{dx}{(8-5x)^3} = \int (8-5x)^{-3}\,dx = -\frac{1}{5}\int u^{-3}\,du = -\frac{1}{5}\cdot -\frac{1}{2}u^{-2}+C$

let $u = 8-5x$

$du = -5\,dx$

$-\frac{1}{5}\,du = dx$

$= \frac{1}{10}(8-5x)^{-2}+C$

8. $\displaystyle\int_0^2 \frac{x}{\sqrt{x^2+4}}\,dx = \int_0^2 (x^2+4)^{-1/2}\,x\,dx = \frac{1}{2}\int_4^8 u^{-1/2}\,du = \frac{1}{2}\cdot 2\cdot u^{1/2}\Big]_4^8$

let $u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

If $x=0$, $u=4$

If $x=2$, $u=8$

$= \sqrt{8}-\sqrt{4} = 2\sqrt{2}-2$

9. $\displaystyle\int_0^{\pi/4} \tan^3(\theta)\sec^2(\theta)\,d\theta = \int_0^1 u^3\,du = \frac{1}{4}u^4\Big]_0^1 = \frac{1}{4}$

let $u = \tan\theta$

$du = \sec^2\theta\,d\theta$

If $\theta = 0$, $u = \tan 0 = 0$.

If $\theta = \frac{\pi}{4}$, $u = \tan \pi/4 = 1$

10. $\displaystyle\int (x^4-5)x^7\,dx = \int (x^4-5)\cdot x^4 \cdot x^3\,dx = \frac{1}{4}\int u(u-5)\,du$

Let $u = x^4-5$; $x^4 = u-5$

$du = 4x^3\,dx$

$\frac{1}{4}\,du = x^3\,dx$

$= \frac{1}{4}\int (u^2-5u)\,du = \frac{1}{4}\left(\frac{1}{3}u^3 - \frac{5}{2}u^2\right)+C$

$= \frac{1}{12}(x^4-5)^3 - \frac{5}{8}(x^4-5)^2 + C$

11. What is wrong with the calculation

$$\int_{-1}^{1} -x^{-2}dx = x^{-1}|_{-1}^{1} = \frac{1}{1} - \frac{1}{-1} = 2.$$

$f(x) = \frac{-1}{x^2}$ is not defined & not continuous at $x=0$ in the interval $[-1,1]$.