1. (A quick refresher from Monday.) Find the derivatives of the functions below.

(a) $y = e^2 + e^{\sin(cx)}$ where c is a constant.

$$\frac{dy}{dx} = 0 + e^{\sin(cx)} \cdot \frac{d}{dx}(\sin(cx)) = e^{\sin(cx)} \cdot (\cos(cx)) \cdot (c)$$
$$= \boxed{c(\cos(cx))e^{\sin(cx)}}$$

$$\frac{d}{dx}[10^x] = \ln(10) \cdot 10^x$$

(b) $y = x^3 \sin(10^x)$

product rule + chain rule

$$y' = 3x^2 \sin(10^x) + x^3 \cdot \cos(10^x) \cdot \ln(10) \cdot 10^x$$
$$= \boxed{3x^2 \sin(10^x) + \ln(10) 10^x x^3 \cos(10^x)}$$

will need chain rule

(c) $w(r) = \ln(\tan^{-1}(r))$

$$w'(r) = \frac{1}{\tan^{-1}(r)} \cdot \frac{d}{dr}(\tan^{-1}(r)) = \left(\frac{1}{\tan^{-1}(r)}\right)\left(\frac{1}{r^2+1}\right)$$
$$= \boxed{\frac{1}{(r^2+1)\tan^{-1}(r)}}$$

(d) $f(x) = x^{\ln(x)}$ Hint: Use logarithmic differentiation (which means to start by taking the natural log of both sides of the equation.)

$$\ln(f(x)) = \ln(x^{\ln x}) = (\ln(x))(\ln(x)) = [\ln(x)]^2$$

take derivative implicitly.

$$\frac{1}{f(x)} \cdot f'(x) = 2 \cdot \ln(x) \cdot \frac{1}{x}$$
$$f'(x) = f(x) \cdot \frac{2\ln(x)}{x} = \boxed{\frac{2\ln(x)\, x^{\ln(x)}}{x}}$$

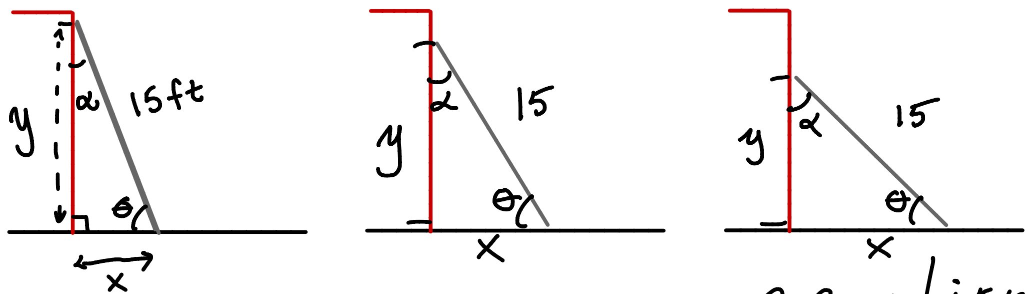
2. Compare $f'(x)/f(x)$ and $f^{50}(x)$ for $f(x) = P_0 e^{kt}$, $f(x) = x^2$ and $f(x) = x^{10}$.

$$f(x) = P_0 e^{kt}$$
$$f'(x) = k P_0 e^{kt} = k f(x)$$
$$f^{(50)}(x) = k^{50} P_0 e^{kt}$$

$$f(x) = x^2$$
$$f'(x) = 2x$$
$$= \frac{2f(x)}{x}$$
$$f^{(50)}(x) = 0$$

$$f(x) = x^{10}$$
$$f'(x) = 10x^9$$
$$= \frac{10 \cdot f(x)}{x}$$
$$f^{(50)}(x) = 0$$

3. Chapter 4 is about applications of the derivative. Section 4.1 is about Related Rate Problems.
Example: A 15-ft ladder is leaning against a wall. The top of the ladder slides down the wall. Assume that the ladder is rigid and does not shorten or lengthen as it slides. Draw a picture. Label with variables the lengths that are changing over time. Label with constants that things that are fixed. Which variables do you expect to have a positive derivative with respect to time? Negative? Zero? What equations can you think of that related some of the variables in your picture?



positive derivative: x, α
negative derivative: y, θ
zero derivative: 15

equations:

(i) $x^2 + y^2 = 15^2$

(ii) $\sin(\theta) = \frac{y}{15}$ $\quad \tan(\theta) = y/x$

$\cos(\theta) = x/15$

$\sin(\alpha) = x/15$ $\quad \tan(\alpha) = x/y$

$\cos(\alpha) = y/15$

- Take derivative implicitly wrt t-time.

(i) $2\overset{+}{x}\overset{+}{\frac{dx}{dt}} + 2\overset{+}{y}\overset{-}{\frac{dy}{dt}} = 0$

(ii) $\overset{+}{\cos(\theta)} \cdot \overset{-}{\frac{d\theta}{dt}} = \frac{1}{15} \cdot \overset{-}{\frac{dy}{dt}}$

- Think about the signs of things.

- What do you need to know to determine $\frac{dy}{dt}$?

(i) $x, \frac{dx}{dt}, y$. (All?)

(ii) $\theta, \frac{d\theta}{dt}$

4. A list of derivative rules you will need to *know* how and when to use.

(a) $\dfrac{d}{dx}\left[f(x)g(x)\right] = f' \cdot g + f \cdot g'$

(b) $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \left(g \cdot f' - f \cdot g'\right) / g^2$

(c) $\dfrac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

(d) $\dfrac{d}{dx}\left[\sin(x)\right] = \cos(x)$

(e) $\dfrac{d}{dx}\left[\cos(x)\right] = -\sin(x)$

(f) $\dfrac{d}{dx}\left[\tan(x)\right] = \sec^2(x)$

(g) $\dfrac{d}{dx}\left[\sec(x)\right] = \sec(x)\tan(x)$

(h) $\dfrac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)$

(i) $\dfrac{d}{dx}\left[\csc(x)\right] = -\csc(x)\cot(x)$

(j) $\dfrac{d}{dx}\left[\sin^{-1}(x)\right] = \dfrac{1}{\sqrt{1-x^2}}$

(k) $\dfrac{d}{dx}\left[\cos^{-1}(x)\right] = \dfrac{-1}{\sqrt{1-x^2}}$

(l) $\dfrac{d}{dx}\left[\tan^{-1}(x)\right] = \dfrac{1}{1+x^2}$

(m) $\dfrac{d}{dx}\left[e^x\right] = e^x$

(n) $\dfrac{d}{dx}\left[a^x\right] = \ln(a)\, a^x$

(o) $\dfrac{d}{dx}\left[\ln(x)\right] = \dfrac{1}{x}$

(p) $\dfrac{d}{dx}\left[\log_a(x)\right] = \dfrac{1}{(\ln a)x}$