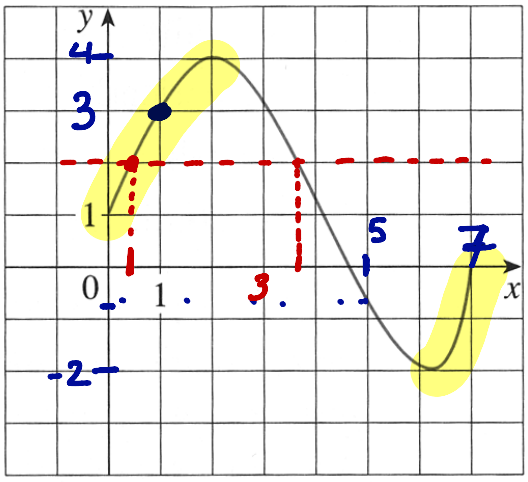
WORKSHEET: REVIEW OF FUNCTIONS

1. The graph of a function f is shown below. Find the following:



a) $f(1)$ and $f(5)$ $= -0.7$ (an estimate) $= 3$

b) the domain of f $[0,7]$

c) the range of f $[-2,4]$

d) For which value(s) of x is $f(x) = 2$?

roughly $x \approx 0.5$, $x \approx 3.7$

e) Where is f increasing? (give x-values)

$(0,2) \cup (6.2,7)$

2. Let $f(x) = 5 - 3x^2$. Find and simplify the following expressions. Are (b) and (c) different?

(a) $f(3) = 5 - 3(3)^2 = 5 - 27 = -22$

(b) $f(a^2) = 5 - 3(a^2)^2 = 5 - 3a^4$

not the same

(c) $[f(a)]^2 = (5 - 3a)^2 = 25 - 30a + 9a^2$

(d) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{5 - 3(x+h)^2 - (5 - 3x^2)}{h} =$

$= \dfrac{5 - 3x^2 - 6xh - h^2 - 5 + 3x^2}{h} = \dfrac{-6xh - h^2}{h} = \dfrac{h(-6x - h)}{h} = -6x - h$

3. Find the domain and range of each of the following functions. Use interval notation.

(a) $f(x) = \dfrac{1}{x^2 - 5}$ (The range is tricky. Look for y-values that are *not* possible.)

domain: all reals except $x = \pm\sqrt{5}$ OR $(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)$

range: $(-\infty, 0) \cup (0, \infty)$ (Since the numerator is always 1, the fraction is never zero.)

(b) $f(x) = \sqrt{11 - x}$

We need $11 - x \geq 0$. So $11 \geq x$.

domain: $(-\infty, 11]$

range: $[0, \infty)$ ← Using the fact that $f(x)$ is just $y = \sqrt{x}$ shifted & flipped.

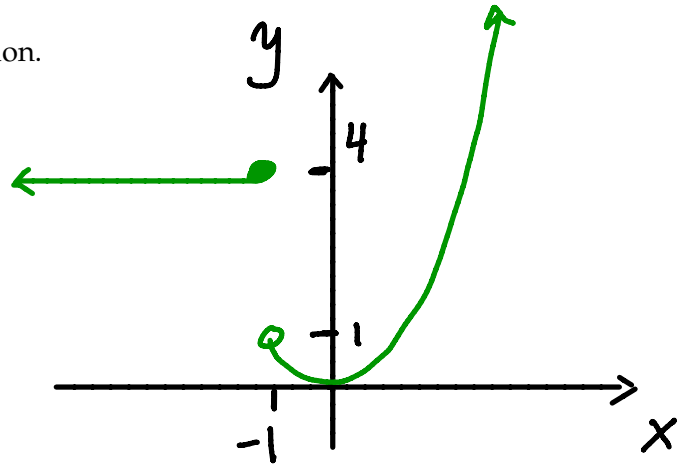
(c) $g(x) = 8.245e^x$

domain: $(-\infty, \infty)$

range: $(0, \infty)$ ← Using the fact that g is just a vertical stretching of $y = e^x$

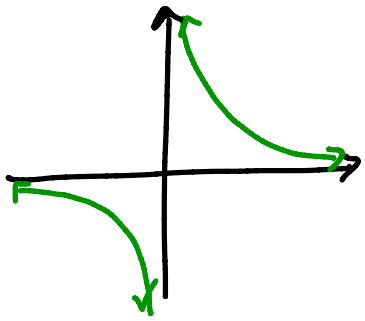
4. Graph the piecewise defined function.

$$f(x) = \begin{cases} 4 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$



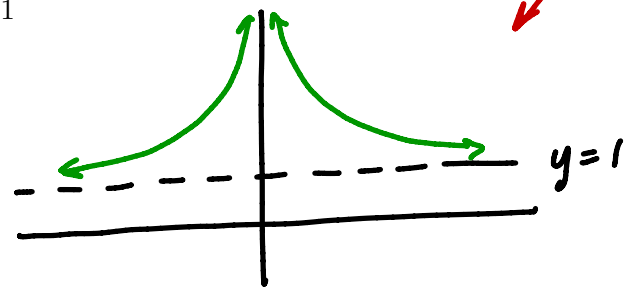
5. Give a rough sketch of each of the following functions. What do you think are the crucial properties to illustrate? What are the important points, if any?

(a) $f(x) = \frac{1}{x}$

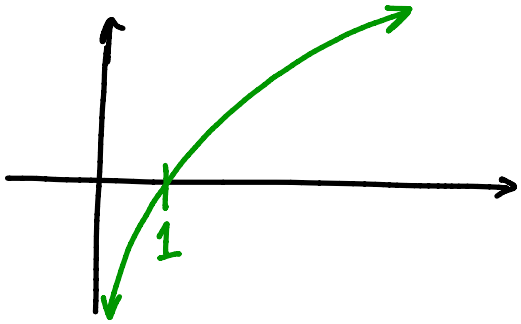


asymptotes + symmetry

(b) $f(x) = \frac{1}{x^2} + 1$

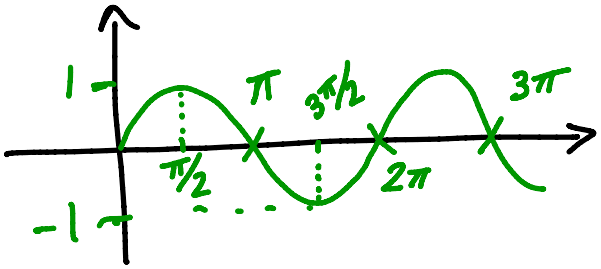


(c) $f(x) = \ln(x)$



asymptote
x-intercept.

(d) $f(x) = \sin(x)$



- repetion
- amplitude (1)
- frequency (2π)
- intercepts.

6. Explain in your own words what is meant by the inverse of the function $f(x)$?

It is a function that undoes f.