1. What does it mean to say that $y = g(x)$ is the inverse of $y = f(x)$? Give some examples of functions f and g that are inverses of each other? How do you know they are inverses?

g undoes f.

$g(f(x)) = x$ and $f(g(x)) = x$.

When you do f and then g, (or vice vesa) the input is unchanged.

Examples

$f(x) = 2x$, $g(x) = \frac{1}{2}x$

$f(x) = x+10$, $g(x) = x-10$

$f(x) = e^x$, $g(x) = \ln x$

$f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x}$

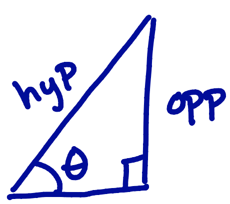
The graphs of g and f are reflections about $y=x$.

Inputs and outputs are switched

$a \to \boxed{f} \to b \qquad b \to \boxed{f^{-1}} \to a$

2. There are three particularly useful ways of thinking about trigonometric functions: (A) sides of a right triangle, (B) points on the unit circle in the xy-plane, (C) as a graph. Can you describe the sine function in each of these ways?

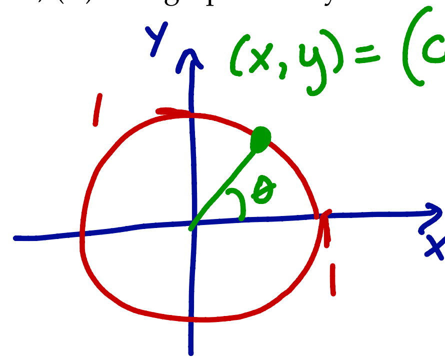
$\boxed{A}$



$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\boxed{B}$

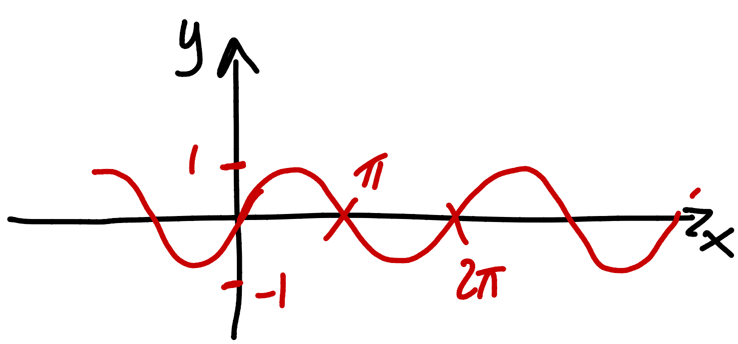
$(x,y) = (\cos\theta, \sin\theta)$



!!

2^{nd} coord.

$\boxed{C}$



What is a radian?

Use input $\leftrightarrow$ output switch.
If $f^{-1}(x)=y$ then $f(y)=x$.

3. You are given the function $f(x)$. Without explicitly finding a formula for $f^{-1}(x)$, find $f^{-1}(1)$ for each function below:

(a) $f(x) = 1 - \sqrt[3]{x+2}$

We want $f^{-1}(1)=\square$.
So $f(\square)=1$.
So $\square=-2$.

Answer: $f^{-1}(1)=-$

(b)

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2.0
$f(x)$	20	10	5	3	2.5	2	1.5	1	0.25

We want $f^{-1}(1)=\square$
So $f(\square)=1$

So look for an f-value of 1.

Since $f(1.75)=1$,
we know $f^{-1}(1)=1.75$

4. Solve each equation below for x.

(a) $10 = 2e^{x+1}$

$5=e^{x+1}$
$\ln(5)=x+1$
$x=(\ln(5))-1$

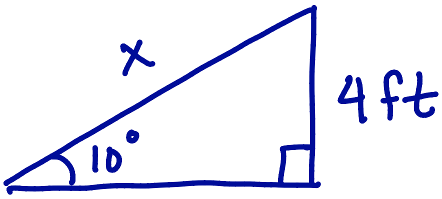
(b) $\ln(x^2-1)=1$

$x^2-1=e^1$
$x=\pm\sqrt{e+1}$

5. What does the previous problem have to do with inverses?

We are using $e^{\ln\square}=\square$ and $\ln(e^{\square})=\square$ to solve.
These properties hold because $f(x)=e^x$ and $g(x)=\ln x$ are inverses.

6. A wooden ramp is to be built with one end on the ground and the other end at the top of a short staircase. If the top of the staircase is 4 ft from the ground and the angle between the ground and the ramp is to be 10^o, how long does the ramp need to be?



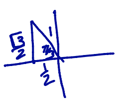
$\frac{4}{x} = \sin(10^\circ)$

So $x = \frac{4}{\sin(10^\circ)}$ ft ≈ 23 ft

7. Convert $2\pi/3$ radians to degrees. $\left(\frac{2\pi}{3} \text{ rad}\right)\left(\frac{180 \text{ deg}}{\pi \text{ rad}}\right) = \frac{2 \cdot 180}{3} = 120^\circ$

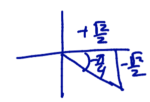
or $\frac{2}{3}(180) = 120^\circ$

8. Without a calculator evaluate:



(a) $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$

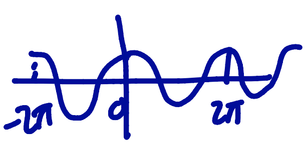
(b) $\cos(\frac{5\pi}{4}) = -\sqrt{2}/2$



(c) $\tan(\frac{-\pi}{4}) = -1$

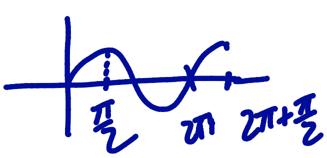
9. Use graphs to solve the equations below.

(a) $\cos x = 1$ $\quad x = 2\pi k$, k integer

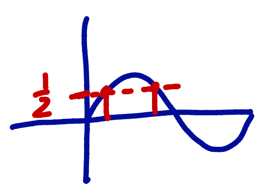


(c) $\tan x = 0$ $\quad x = \pi k$, k integer

(b) $\sin x = 1$ $\quad x = \frac{\pi}{2} + 2\pi k$, k integer



(d) $\sin x = 1/2$ (Find all solutions in $[0, 2\pi]$.) $\quad x = \frac{\pi}{6}, \frac{5\pi}{6}$



10. Find the equation of the line between the points $(-1, 2)$ and $(3, 6)$.

Slope: $m = \frac{\Delta y}{\Delta x} = \frac{6-2}{3--1} = \frac{4}{4} = 1$

line: $y - 2 = 1(x+1)$ or $y = 3 + x$

11. Assume $P(t) = \sqrt{4t+4} - 2$ gives the distance traveled by a runner in the first 30 seconds of a race where t is measured in seconds and P is measured in meters. (So the domain of P is $[0, 30]$.)

~~(a) Is this a plausible model (given the very brief description)? Why or why not?~~

points

$(3,2)$

$(15,6)$

$P(3) = \sqrt{4 \cdot 3 + 4} - 2 = \sqrt{16} - 2 = 2$

$P(15) = \sqrt{4 \cdot 15 + 4} - 2 = \sqrt{64} - 2 = 8 - 2 = 6$

a ~~(b)~~ Find the slope, m, of the line between the points $(3, P(3))$ and $(15, P(15))$.

$m = \frac{\Delta P}{\Delta t} = \frac{6-2}{15-3} = \frac{4}{12} = \frac{1}{3}$

b ~~(c)~~ What should the units of m be and why? What does the slope mean in the context of the problem?

units of m = $\frac{\text{units of P}}{\text{units of t}} = \frac{\text{meters}}{\text{second}} = m/s$

In the first 30 seconds, the runner had an average velocity of $\frac{1}{3}$ m/s.