REVIEW FOR FINAL EXAM

Logistics

The final exam is Wednesday April 27 10:15-12:15 in Chapman 104.

All you need to take the exam is a writing utensil. Scratch paper or extra paper will be provided for you, if needed. Books, notes and calculators are not allowed. There will be no problems the require (or need) a calculator.

Topics

Section 2.1

Secant lines and tangent lines. Average velocity and instantaneous velocity. Average rate of change and instantaneous rate of change.

Example: Sketch the graph $y = x^3 + 1$. Find the secant line between the points on the graph where $x = 1$ and $x = 3$. Sketch the secant line on the graph. Find an equation of the tangent line to the graph at $x = 1$ and sketch it on the graph. (NOTE: We get to answer the second part of this question using our knowledge of the derivative!)

Section 2.2

One-sided and two-sided limits from a graph. Vertical asymptotes and limits.

Example: Sketch a graph with *all* of the following properties:

- $f(x)$ is defined for all real numbers. (ie its domain is $(-\infty, \infty)$.
- $\lim_{x\to1^-} f(x) = 0$, $\lim_{x\to1^+} f(x) = 4$, $f(1) = 4$
- $\lim_{x\to3} f(x) = 4$, $f(4) = -1$.
- $\lim_{x\to-1^-} f(x) = \infty$

Section 2.3

Evaluating limits algebraically.

Example: Evaluate $\lim_{x\to9} \frac{3-\sqrt{x}}{9-x}$ and $\lim_{x\to1/2^+} \frac{4x^2-18x}{2x-1}$

Section 2.4

Continuity. From a graph, determine where a graph is or is not continuous. From an algebraic description of a function, determine where a function is or is not continuous. Be able to explain why a function is not continuous at a point. The Intermediate Value Theorem.

Example: Look at your graph from Section 2.2. Where does it fail to be continuous and why? Where are the functions $f(x) = \frac{3-\sqrt{x}}{9-x}$ and $g(x) = \frac{4x^2-18x}{2x-1}$ continuous?

Section 3.1

The relationship between secant lines and the derivative.

Example: Explain what the expression $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ means in terms of secant lines, tangent lines and the derivative. Draw a picture to illustrate you idea.

Section 3.2

The derivative as a function. The formal definition of the derivative. The relationship between the graph of $f(x)$ and the graph of $f'(x)$.

Example: Sketch the derivative of your graph from the Section 2.2 example. Use the definition of the derivative to find $f'(x)$ for $f(x) = 1/x^2$.

Section 3.3

Derivative rules: power, constant, sum/difference, product, quotient.

Example: Find the derivative of $y = 2x^{0.05} - \frac{x}{10}$ and $f(x) = \frac{x^3}{1-x}$

Section 3.4

The derivative as a rate of change. Interpretations of the derivative. Velocity and acceleration.

Example: Assume the distance traveled by a snow machine on a straight trail is given by $s(t)$ where t is in hours starting at 12 noon and s is in miles. Interpret $s'(4) = 10$. Interpret $s(4) - s(0)$. Interpret $(s(4) - s(1))/(4 - 1)$. Using the fact that that $s''(4) = -1.2$, estimate $s'(4.5)$.

Sections 3.5-3.9

Techniques and rule for taking derivative

Section 4.1

Related Rate Problems. All of these problems are word problems asking for a rate of change of some quantity with respect to time.

Example: An airplane is flying overhead at a constant elevation of 4000 feet as it passes directly over a man standing on the ground. If the plane is flying at a speed of 600 feet per second, how fast is the plane moving away from the man 5 seconds after it passes over his head? Assume the plane is flying in a straight line.

Section 4.2

Linear Approximations and Differentials. These problems ask you to find the linear approximation or differential of a function for particular values and then use these things (the linear approximation or differential) to estimate other things.

Example: Find the linear approximation of $f(x) = 5\sin(x)$ when $a = 0$ and use it to estimate $5\sin(-0.1)$

Example: Find the differential of $f(x) = 4\sqrt{x}$ when $x = 9$ and use it to estimate how much f will change if x changes from 9 to 9.01

Section 4.3

Maxima and minima. Absolute and local. Critical points. These problems are of two types: Finding ABSOLUTE extrema on closed-bounded intervals and finding local extrema in general.

Example: Find the absolute maximum and the absolute minimum of $f(x) = x^2 - 3x^{2/3}$ on $[0, 8]$.

Example: Identify any local extrema of $y = x^2 - \frac{1}{x^2}$.

Section 4.5

Derivatives and the Shape of a Graph. These problems ask you to use f' and f'' to determine when the original function, f, is increasing or decreasing, concave up or concave down, has extrema, has inflection points, and to draw sophisticated graphs.

Example: Draw some not-too-complicated graph. Now assume it is f'. What can you say about the graph of f?

Example: If $f' > 0$ for $x > 0$, $f' < 0$ for $x < 0$, $f'' > 0$ for $-2 \leq x \leq 2$ and $f'' < 0$ for $x < -2$ and for $x > 2$, sketch f.

Section 4.6

Limits at Infinity and Asymptotes. The problems either ask yo to evaluate a limit as $x \to \pm\infty$ or ask to find and justify the existence of a horizontal asymptote.

Example: Determine if the graph of $f(x) = \frac{3x^3 - e^x}{2x^3}$ has a horizontal asymptote. Justify your answer.

Section 4.7

Optimization Problems. These are word problems where you are asked to maximize or minimize some quantity. Crucial steps here include
(a) identify the quantity to be maximized/minimized,
(b) write the quantity from part (a) as a function of one variable,
(c) identify the domain of the function from part (b),
(d) take derivative and find critical points for function from part (b),
(e) check/justify that your cp actually corresponds to a max/min,
(f) answer the question.

Example: Go work problems from old midterms.

Section 4.8

L'Hopital's Rule. Be able to use L'Hopital's Rule to evaluate limits of a variety of indeterminate forms.

Example: Evaluate $\lim_{x \to 0^+} x \ln(x^4)$

Section 4.10

Antiderivatives and Initial Value Problems

Example: Evaluate $\int (\frac{3}{sqrtx} - \csc^2(x))\, dx$

Example: If an object as acceleration $a(t) = x + \sin(x)$, find its velocity equation assuming $v(0) = 10$.

Section 5.1

Approximating areas. Use rectangles with left- or right-hand endpoints to estimate the area under a curve.

Example: Use L_8 (ie eight rectangles with left-hand endpoints) to estimate the area under $y = \sqrt{x}$ on the interval $[0, 4]$. No need to get a decimal approximation. (!!)

Section 5.2

The Definite Integral as Signed Area under a Curve.

Example: Sketch the graph of $y = 10 - 5x$. Use this graph to evaluate $\int_1^6 (10 - 5x)\, dx$.

Section 5.3

The Fundamental Theorem of Calculus, parts I and II.

Example: Find the derivative of the function $F(x) = \int_1^{\cos(x)} (1 - t^2)\, dt$

Example: Evaluate $\int_1^5 \frac{x}{1+x^4}\, dx$

$\boxed{\text{Section 5.4}}$
The Net Change Theorem

Example: If $v(t)$ is the velocity of a car along a straight road in miles per hour, interpret the meaning of $\int_1^5 v(t)\; dt = -20$. Assume 1 and 5 are measured in hours.

$\boxed{\text{Section 5.5}}$
The method of substitution. (See the second example from Section 5.3 above.)

$\boxed{\text{Sections 5.6-5.7}}$
More integration formulas including those of exponential functions, logarithms, and inverse trigonometric functions.