Review for Final Exam

Topics from Chapter 5:

- §5.1 & 5.2: Approximating Area and the Definite Integral
- §5.3: The Fundamental Theorem of Calculus
- §5.4: The Net Change Theorem
- §5.4-5.7: Integration Formulas and the Method of Substitution

$\frac{15}{12}$

1. Compute the integrals below.

   (a) $\displaystyle\int_{-1}^{0}(t^{1/3}-t^{2/3})dt$ $= \dfrac{t^{4/3}}{\frac{4}{3}} - \dfrac{t^{5/3}}{\frac{5}{3}}\Bigg|_{-1}^{0} = \dfrac{3}{4}t^{\frac{4}{3}} - \dfrac{3}{5}t^{5/3}\Bigg]_{-1}^{0}$

   $= (0) - \left(\frac{3}{4}(-1)^{4/3} - \frac{3}{5}(-1)^{5/3}\right) = -\frac{3}{4} - \frac{3}{5} = \frac{-27}{20}$

   (b) $\displaystyle\int_{0}^{2} x\sqrt{4-x^2}\,dx$ $= \displaystyle\int_4^0 u^{1/2}\left(-\tfrac{1}{2}du\right) = -\frac{1}{2}\int_0^4 u^{\frac{1}{2}}du = -\frac{1}{2}\cdot\frac{u^{3/2}}{\frac{3}{2}}\Bigg]_0^4 = -\frac{1}{2}\cdot\frac{2}{3}\cdot u^{3/2}\Bigg]_0^4$

   let $u = 4-x^2$ | If $x=0$, $u=4$
   $du = -2x\,dx$ | $x=2$, $u=0$
   $-\frac{1}{2}du = x\,dx$

   $= -\frac{1}{3}\left(4^{3/2} - 0^{3/2}\right) = -\frac{8}{3}$

   (c) $\displaystyle\int (x^{2.35} + \frac{3}{4x} + e^x)\,dx$

   $= \displaystyle\int\left(x^{2.35} + \frac{3}{4}\cdot\frac{1}{x} + e^x\right)dx = \frac{x^{3.35}}{3.35} + \frac{3}{4}\ln|x| + e^x + C$

   (d) $\displaystyle\int \frac{1}{1+4x^2}\,dx$ $= \displaystyle\int\frac{1}{1+(2x)^2}dx = \frac{1}{2}\int\frac{1}{1+u^2}du = \frac{1}{2}\arctan(u) + C$

   let $u=2x$
   $du = 2\,dx$
   $\frac{1}{2}du = dx$

   $= \frac{1}{2}\arctan(2x) + C$

   (e) $\displaystyle\int \sec^2(5x) + e^{3x}\,dx$

   $= \frac{1}{5}\tan(5x) + \frac{1}{3}e^{3x} + C$

2. Find and simplify the derivative of the function $h(x) = \int_1^{e^x} x^7 \ln(x)\,dx$

$$\frac{d}{dx}\int_1^{e^x} x^7 \ln(x)\,dx = (e^x)^7 \ln(e^x)\cdot e^x = e^{7x}\cdot x\cdot e^x = xe^{8x}$$

3. A population of chickadees is changing at a rate of $r(t)$ chickadees per year.

(a) What does $\int_1^4 r(t)\,dt = 400$ mean? Make sure to include units in your answer.

Between years 4 and year 1, the net change in chickadee population was 400 chickadees.

(b) Is it possible for $\int_0^{t_0} r(t)\,dt < 0$ for some time $t_0 > 0$? Explain your answer.

Yes. It would indicate a net loss in the chickadee population over the first t_0 years.

(c) Evaluate $\int_1^4 (5r(t) + 10)\,dt$ $= 5\int_1^4 r(t)\,dt + \int_1^4 10\,dt$

$= 5(400) + 10\cdot(4-1) = 2000 + 30$

$= 2030.$

A quick review of main ideas/strategies.

- (§4.7) Optimization
- (§4.3 & 4.5) Derivatives, the Shape of a Graph, and Extrema
- (§4.6 & 4.8) Limits, Asymptotes, and L'Hopital's Rule
- (§4.1) Related Rate Problems
- (§4.10) Initial Value Problems
- (§4.2) Linear Approximations and Differentials

4. A particle is moving with acceleration $a(t) = t + e^{t/2}$ in meters per second per second. You measure that at time $t = 0$, its position is given by $s(0) = 0$ meters and its velocity is given by $v(0) = 8$ meters per second. Determine the position of the particle at time $t = 1$.

$a(t) = t + e^{t/2}$

$v(t) = \int (t + e^{t/2})\,dt$

$= \frac{1}{2}t^2 + 2e^{t/2} + C$

$v(0) = 8 = \frac{1}{2}(0)^2 + 2e^0 + C$

So $8 = 2 + C$. So $C = 6$.

$v(t) = \frac{1}{2}t^2 + 2e^{t/2} + 6$

$s(t) = \int \left(\frac{1}{2}t^2 + 2e^{t/2} + 6\right) dt$

$= \frac{1}{6}t^3 + 4e^{t/2} + 6t + C$

$s(0) = 0 = 0 + 4e^0 + 0 + C$

So $0 = 4 + C$. So $C = -4$

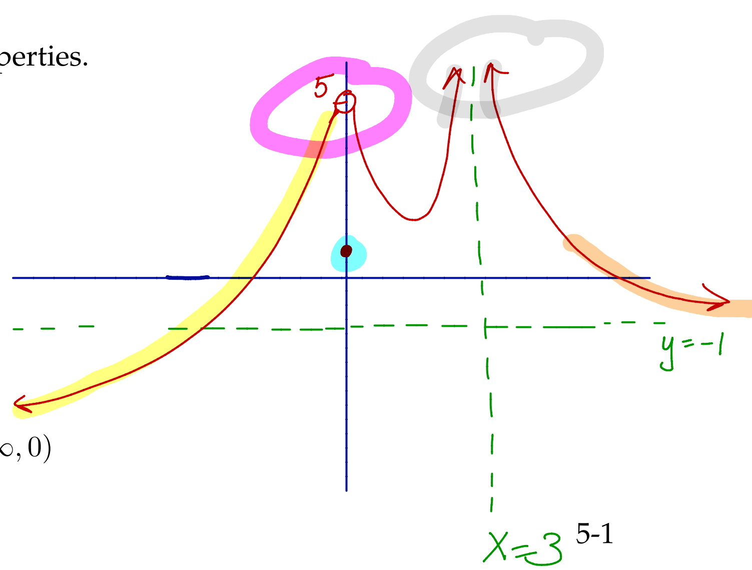
$s(t) = \frac{1}{6}t^3 + 4e^{t/2} + 6t - 4.$

Position when $t=1$:

$s(1) = \frac{1}{6} + 4e^{1/2} + 6 - 4$

$= \frac{13}{6} + 4e^{\frac{1}{2}}$ meters

5. Sketch a graph $H(x)$ with all of the following properties.



- The domain of $H(x)$ is $(-\infty, 3) \cup (3, \infty)$
- $H(0) = 1$ point $(0,1)$
- $\lim_{x\to 0} H(x) = 5$ close to $x=0$, y is @ 5.
- $\lim_{x\to\infty} H(x) = -1$ H.A @ $y=-1$
- $\lim_{x\to 3} H(x) = \infty$ V.A @ $x=3$
- $H'(x) > 0$ and $H''(x) > 0$ on the interval $(-\infty, 0)$

H is ↑ H is ∪

6. The height of a right circular cylinder is increasing at a rate of 2 meters per second while its volume remains constant. At what rate is the radius changing when the radius is 10 meters and height is 20 meters. (Note, the volume of a cylinder is given by $V = \pi r^2 h$ where r is the radius and h is the height of the cylinder.)

Info from problem statement

$\frac{dh}{dt} = 2$ m/s

$\frac{dV}{dt} = 0$ m/s

Find $\frac{dr}{dt}$ when $r=10$, $h=20$

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}\cdot h + r^2\frac{dh}{dt}\right)$

$0 = \pi\left(2\cdot 10\cdot\frac{dr}{dt}\cdot 20 + (10)^2(2)\right)$

$0 = 400\frac{dr}{dt} + 200$

So $\frac{dr}{dt} = \frac{-200}{400} = -\frac{1}{2}$ m/s

7. Write the equation of the tangent line to $f(x) = \sqrt{x}$ when $x = 16$ and use it to estimate $\sqrt{16.1}$.

$f(x) = x^{1/2}$ $\quad f(16) = 4$ $\quad y - 4 = \frac{1}{8}(x-16)$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(16) = \frac{1}{8} = 0.125$ $\quad$ or $y = 4 + \frac{1}{8}(x-16)$

8. Let $f(x) = \sqrt{x}$.

(a) Write the linearization of $f(x)$ at $a = 16$.

(same problem as above ↑)

$L(x) = 4 + \frac{1}{8}(x-16)$ or $L(x) = 4 + 0.125(x-16)$

(b) Use the linearization in part (a) to estimate $\sqrt{16.1}$

$L(16.1) = 4 + 0.125(16.1-16) = 4 + (0.125)(0.1)$

$= 4 + 0.0125 = 4.0125$

(c) Would the linearization from part (a) give a good estimate of $\sqrt{7}$? Explain.

Probably not. $x=7$ is not close to $x=16$, the point of tangency.