option1 : L'Hop

9. Find the following limits

form $\frac{0}{0}$ (a) $\displaystyle\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5} \overset{H}{=} \lim_{x\to 5} \frac{-x^{-2}}{1} = \frac{-1}{25}$

option2: algebra $\displaystyle\lim_{x\to 5} \left(\frac{1}{x-5}\right)\left(\frac{5-x}{5x}\right) = \lim_{x\to 5} \frac{-(x-5)}{5x(x-5)} = \lim_{x\to 5} \frac{-1}{5x} = \frac{-1}{25}$

opt1: algebra

form $\frac{\infty}{\infty}$ (b) $\displaystyle\lim_{x\to\infty} \frac{2x^2-3}{4+5x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2-\frac{3}{x^2}}{\frac{4}{x^2}+5} = \frac{2}{5}$

opt2: L'Hop

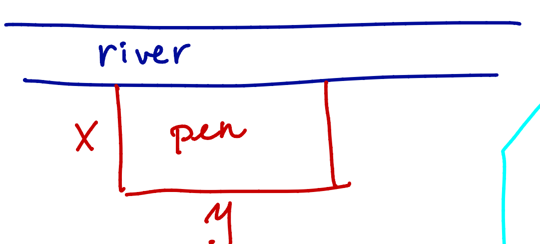
$\displaystyle\overset{H}{=} \lim_{x\to\infty} \frac{4x}{10x} = \lim_{x\to\infty} \frac{4}{10} = \frac{4}{10} = \frac{2}{5}$

(c) $\displaystyle\lim_{x\to 0} \frac{x^2}{1-\cos(x)} \overset{H}{=} \lim_{x\to 0} \frac{2x}{\sin(x)} \overset{H}{=} \lim_{x\to 0} \frac{2}{\cos(x)} = \frac{2}{1} = 2$

form $\frac{0}{0}$

form $\frac{0}{0}$

10. A farmer has 400 meters of fencing and wants to fence off a rectangular field that borders a straight river. No fencing is needed along the river, which forms one side of the rectangle. What are the dimensions of the field that has the largest area?



goal: maximize area

$A = xy$ ← need to be a fcn of 1 variable.

use $400 = 2x + y$

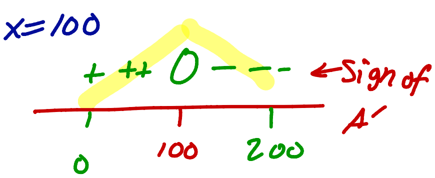
or $y = 400 - 2x$

plug in:

$A(x) = x(400-2x) = 400x - 2x^2$

$A'(x) = 400 - 4x = 0$. So $x = 100$

Justify : opt1 : (1st der. Test)



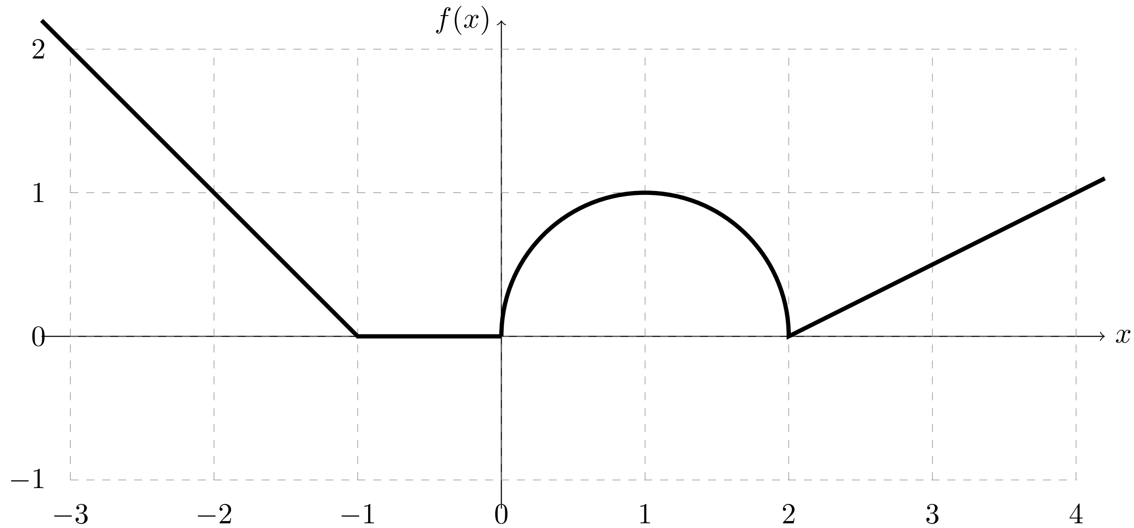
opt2 : (2nd der. test)

$A'' = -4 < 0$ So A is cc down: $\frown$

In both cases $A(x)$ has a max at $x = 100$.

ANSWER: Dimensions $x = 100\,m$, $y = 200\,m$.

11. Consider the function $f(x)$ graphed below. Between $x = 0$ and 2, the graph is of a semicircle of radius 1.



(a) At what x values, if any, does $f'(x)$ not exist?

(at corners) $x = -1, 0, 2$

(b) What is the value of $f'(-2)$?

$f'(-2)$ is slope of f at $x=-2$. So $m = f'(-2) = -1$.

(c) Evaluate $\int_{-1}^{4} f(x)\,dx$. = signed area under $f(x)$.

So $0 + \frac{\pi \cdot (1)^2}{2} + \frac{1}{2}(2)(1) = \frac{\pi}{2} + 1$

(d) Let $g(x) = \int_{1}^{x} f(s)\,ds$. What is the value of $g(0)$?

$g(0) = \int_1^0 f(s)ds = -\int_0^1 f(s)ds = -\left(\frac{1}{4}\pi\right) = \frac{-\pi}{4}$

(e) For $g(x)$ from part **d.**, what is the value of $g'(4)$.

$g'(x) = f(x)$ by FTC part 1

So $g'(4) = f(4) = 1$.