SECTION 2-2: THE LIMIT OF A FUNCTION

1. DEFINITION: **two-sided limit**

   Notation:

   Words:

   It means:

2. Example 1: Evaluate $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ numerically.

3. Example 2: Evaluate $\lim_{x \to 0} \dfrac{\sin(x)}{x}$ numerically.

4. Example 3: Limits do not always exist. Evaluate each limit below numerically and explain why the limits do not exist.

   (a) $\lim_{x \to 2} \dfrac{|x - 2|}{x - 2}$

   (b) $\lim_{x \to 0} \dfrac{1}{x^2}$

It is possible to have one-sided limits. Example 3a at the bottom of page 1 illustrates this.

5. Notation:

$x \to 2^-$ means

and

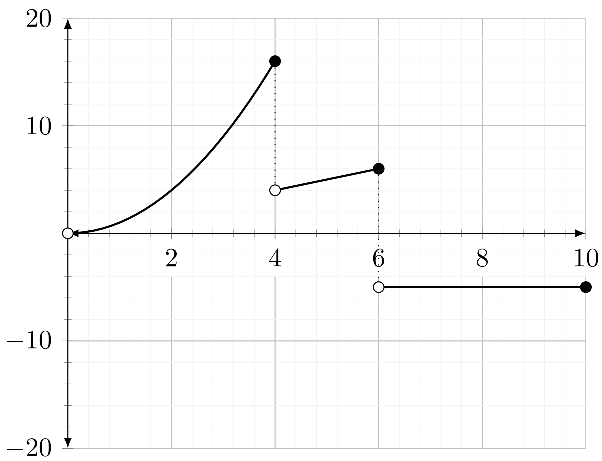
$x \to 2^+$ means

(a) $\displaystyle\lim_{x\to 2^-} \frac{|x-2|}{x-2} =$

(b) $\displaystyle\lim_{x\to 2^+} \frac{|x-2|}{x-2} =$

Limits can also be evaluated graphically.

6. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\displaystyle\lim_{x\to 4^-} g(x) =$ ____________

(b) $\displaystyle\lim_{x\to 4^+} g(x) =$ ____________

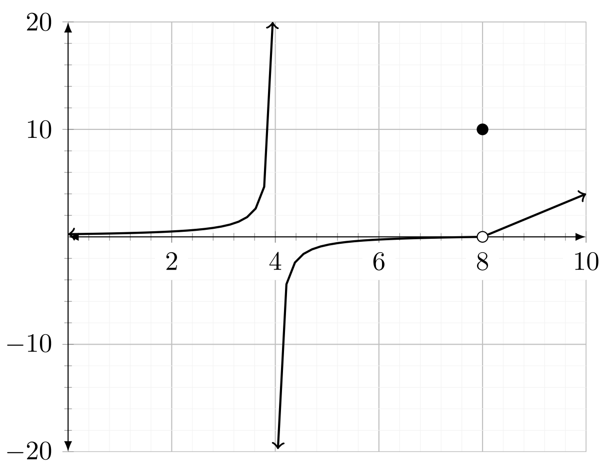
(c) $\displaystyle\lim_{x\to 4} g(x) =$ ____________

(d) $g(4) =$ ____________

(e) $\displaystyle\lim_{x\to 8} g(x) =$ ____________

(f) $g(8) =$ ____________

7. The function $h(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\displaystyle\lim_{x\to 4^-} h(x) =$ ____________

(b) $\displaystyle\lim_{x\to 4^+} h(x) =$ ____________

(c) $\displaystyle\lim_{x\to 4} h(x) =$ ____________

(d) $h(4) =$ ____________

(e) $\displaystyle\lim_{x\to 8} h(x) =$ ____________

(f) $h(8) =$ ____________

8. Find any vertical asymptotes of $f(x) = \frac{2}{x+5}$ and *justify* your answer using a limit.

9. Sketch the graph of an function that satisfies *all* of the given conditions. Compare your answer with that of your neighbor.

$$\lim_{x\to 0^-} f(x) = 1 \quad \lim_{x\to 0^+} f(x) = -2 \quad \lim_{x\to 4^-} f(x) = 3 \quad \lim_{x\to 4^+} f(x) = 0$$

$$f(0) = -2 \qquad f(4) = 1$$