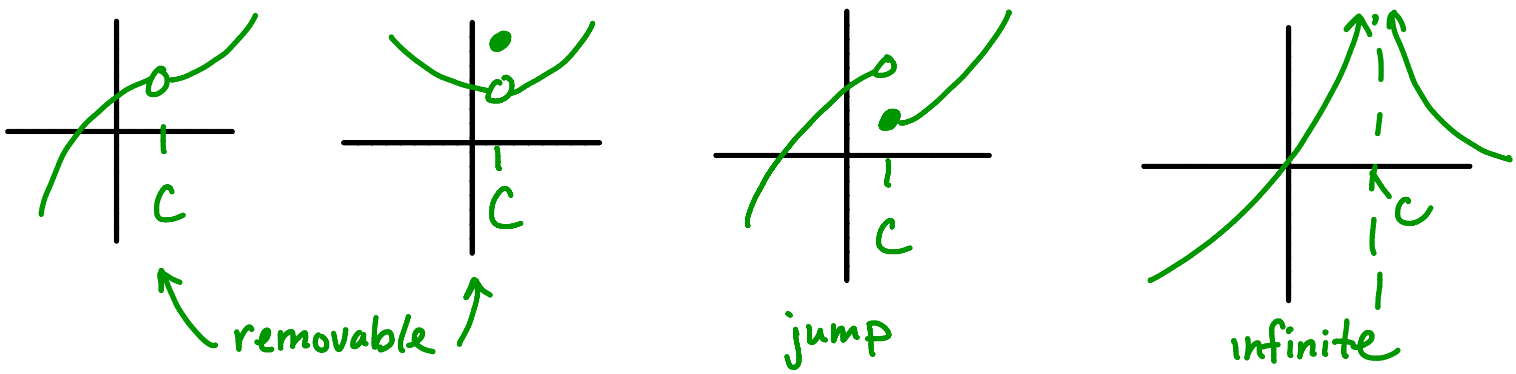
Read Section 2.4. Work the embedded problems.

1. Pictures of graph discontinuities



2. Definition of continuity at a point

Short version: $f(x)$ is continuous at $x=c$ if $\lim_{x\to c} f(x) = f(c)$

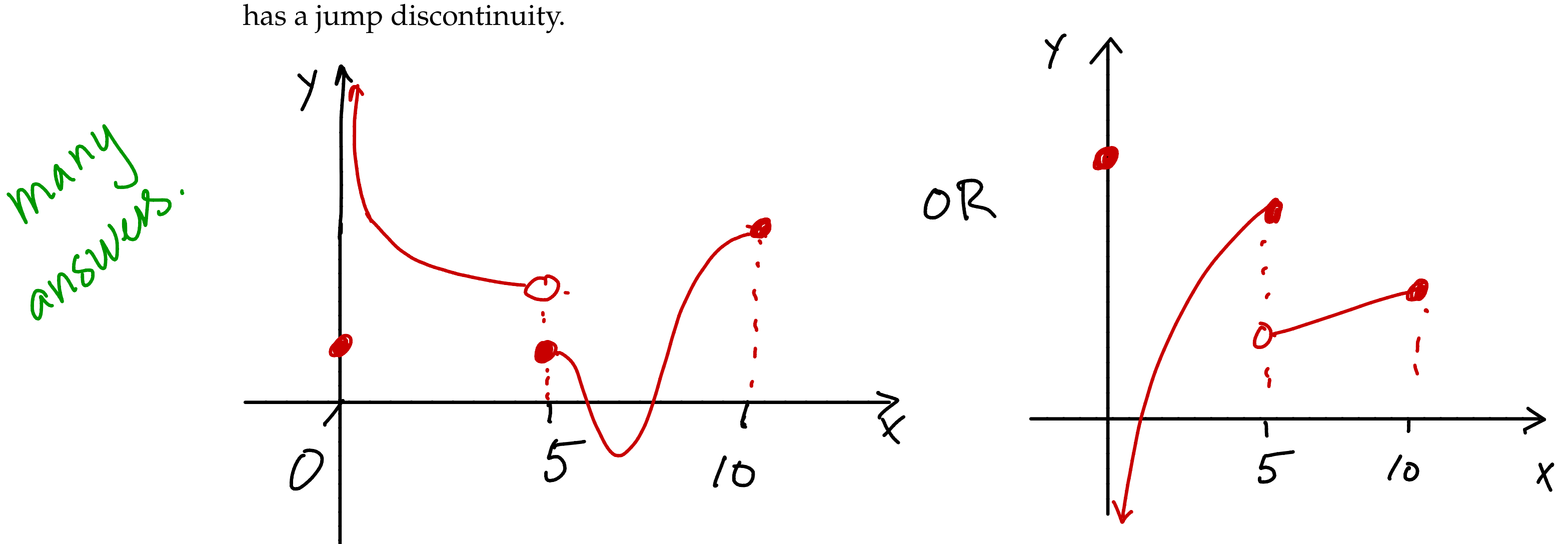
Check list version: $f(x)$ is continuous at $x=c$ if

- ☐ $\lim_{x\to c} f(x)$ exists
- ☐ $f(c)$ exists
- ☐ The previous $\boxed{\text{two numbers}}$ are equal.

3. Sketch the graph of a function $f(x)$ with the following properties:

   (a) the domain of $f(x)$ is the interval $[0, 10]$.

   (b) $f(x)$ is continuous except at $x = 0$ where it has in infinite discontinuity and $x = 5$ where it has a jump discontinuity.



4. Determine the point(s), if any, at which the function $h(x) = \frac{x+2}{x^2-4}$ is discontinuous. Justify your answer. Classify any discontinuity as jump, removable, infinite, or other.

Answers:

discontinuity	type
$x = 2$	infinite
$x = -2$	removable

Work/justification.

$x = 2$: $\lim\limits_{x \to 2^+} \frac{x+2}{x^2-4} = +\infty$ because as $x \to 2^+$, $x+2 \to 4$ (positive) and $x^2-4 \to 0^+$ (also positive).

$x = -2$: $\lim\limits_{x \to -2} \frac{x+2}{x^2-4} = \lim\limits_{x \to -2} \frac{\cancel{x+2}}{\cancel{(x+2)}(x-2)} = \lim\limits_{x \to -2} \frac{1}{x-2} = \frac{-1}{4}$ but $h(-2)$ does not exist.

5. Find the value(s) of k that makes the function continuous over the given interval.

$$f(x) = \begin{cases} e^{kx} & \text{if } 0 \le x < 4 \\ 2x+1 & \text{if } 4 \le x \le 10 \end{cases}$$

We need $e^{kx} = 2x+1$ when $x = 4$.

So solve $\boxed{e^{k \cdot 4} = 2 \cdot 4 + 1}$ for k.

$e^{4k} = 9$

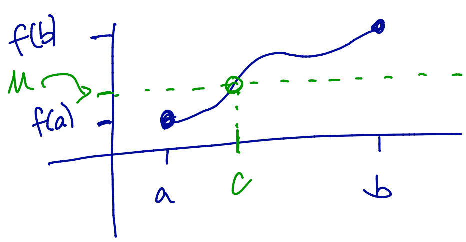
$4k = \ln 9$

$k = \frac{\ln 9}{4}$

6. The Intermediate Value Theorem

If $f(x)$ is continuous on $[a,b]$ and M is a y-value between $f(a)$ and $f(b)$, then there is an x-value c in the open interval (a,b) so that $f(c) = M$.

[Nutshell: If $f(x)$ is continuous, it can't skip over values!]



BONUS:

7. Use the Intermediate Value Theorem to show that the equation $x^4 + x - 3 = 0$ must have a solution in the interval from $x = 1$ to $x = 2$.

① $f(x) = x^4 + x - 3$ is continuous. So I.V.Thm applies.

② $f(1) = 1^4 + 1 - 3 = -1 < 0$ and $f(2) = 2^4 + 2 - 3 = 15 > 0$.

③ Since f is below zero when $x=1$ and greater than zero when $x=2$, f must be exactly zero somewhere between $x=1$ and $x=2$.

Draw your conclusion!