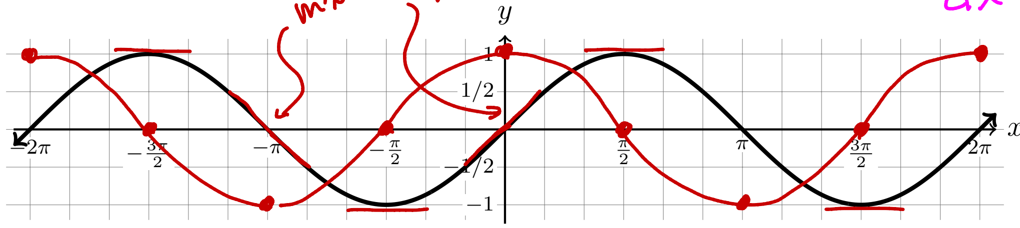
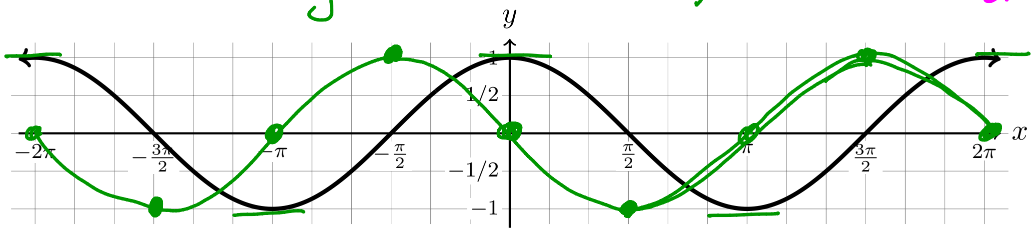
1. (Review from Friday.) On the same set of axes, use the graphs of $f(x) = \sin(x)$ and $g(x) = \cos(x)$ (below) to sketch the graph of their derivatives $f'(x)$ and $g'(x)$.

$f(x) = \sin(x)$ $m \approx -1$ $m \approx 1$ $f'(x) = \cos(x)$ $\frac{d}{dx}[\sin(x)] = \cos(x)$



$g(x) = \cos(x)$ $g'(x) = -\sin(x)$ $\frac{d}{dx}[\cos(x)] = -\sin(x)$



2. Use the definition to find the derivative of $H(x) = x^2$.

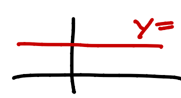
$$H'(x) = \lim_{h\to 0} \frac{H(x+h) - H(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} 2x + h = 2x + 0 = 2x$$

$\frac{d}{dx}[x^2] = 2x$

$\frac{d}{dx}[c] = 0$

3. If $f(x) = 10$, what should $f'(x)$ be and why?

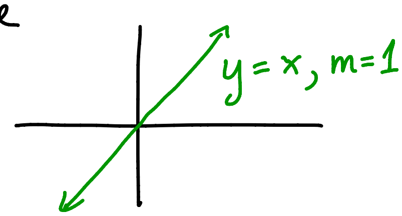


$f'(x) = 0$, b/c f is a horizontal line

4. If $f(x) = c$, where c is some real number, what is $f'(x)$?

$f'(x) = 0$ b/c f is a horizontal line

5. If $f(x) = x$, what should $f'(x)$ be and why?



$f'(x) = 1$, b/c f is a line w/ slope 1.

6. What about $f(x) = 5x$? Explain.

$f'(x) = 5$ b/c f is a line w/ slope 5.

7. What about $f(x) = 5x + 10$? Explain.

$f'(x) = 5$ b/c f is a line w/ slope 5, just shifted up

$$\frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2}x^{-\frac{1}{2}}$$

8. In the 3.2 notes on the definition of the derivative, we found that if $f(x) = \sqrt{x+5} = (x+5)^{1/2}$, then its derivative was:

$$f'(x) = \frac{1}{2\sqrt{x+5}} = \frac{1}{2}(x+5)^{-\frac{1}{2}}$$

Use this to determine the derivative of $g(x) = \sqrt{x}$.

$$g(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

* Go back and look at notation

9. The Power Rule

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

10. The Sum (and Difference) Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

11. The Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

12. Apply the rules to find the derivatives of the functions below. Simplify your answers and write with positive exponents.

(a) $f(x) = e^3$ $\quad f'(x) = 0$

(b) $f(x) = x^{-4}$ $\quad f'(x) = -4x^{-5}$

(c) $H(x) = 4x^{3/2} + 15$ $\quad H'(x) = 4\left(\frac{3}{2}x^{\frac{3}{2}-1}\right) + 0 = 6x^{\frac{1}{2}}$

(d) $j(x) = \frac{\sqrt{2}}{2} + x - 8x^{2.3}$ $\quad j'(x) = 0 + 1 - 8\left(2.3x^{2.3-1}\right)$

$$= 1 - 18.4x^{1.3}$$

13. Find examples of $f(x)$ and $g(x)$ that demonstrate that the rules below are WRONG.

INCORRECT: If $H(x) = f(x)g(x)$, then $H'(x) = f'(x)g'(x)$.

If $f(x) = x$ and $g(x) = x$, then $H(x) = x^2$. So $H'(x) = 2x$.
But $f'(x) \cdot g'(x) = 1 \cdot 1 = 1$

INCORRECT: If $H(x) = \frac{f(x)}{g(x)}$, then $H'(x) = \frac{f'(x)}{g'(x)}$.

If $f(x) = g(x) = x$, then $H(x) = \frac{x}{x} = 1$. So $H'(x) = 0$. But $\frac{f'}{g'} = \frac{1}{1} = 1$.

14. Product Rule $\frac{d}{dx}[f(x)\,g(x)] =$ $f'(x)g(x) + f(x)\cdot g'(x)$

15. Example: Find the derivative of $p(x) = \overbrace{x^2 \sin(x)}^{f \cdot g}$

$$P'(x) = \overbrace{(2x)}^{f'} \cdot \overbrace{(\sin x)}^{g} + \overbrace{x^2}^{f} \cdot \overbrace{(\cos x)}^{g'} = 2x\sin(x) + x^2\cos(x)$$

16. Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ $\dfrac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$

17. Example: Use the Quotient Rule to find the derivative of $p(t) = \frac{\overbrace{\cos(t)}^{f}}{1-2t}$.

$$p'(t) = \frac{\overbrace{(1-2t)}^{g} \cdot \overbrace{(-\sin(t))}^{f'} - \overbrace{\cos(t)}^{f} \cdot \overbrace{(0-2)}^{g'}}{\underbrace{(1-2t)^2}_{}} = \frac{(2t-1)\sin(t) + 2\cos(t)}{(1-2t)^2}$$

g

18. Notation

$y = f(x)$

derivative: y', $f'(x)$, $\frac{dy}{dx}$, $\frac{df}{dx}$, $\frac{d}{dx}[f(x)]$, $\frac{d}{dx}[y]$

19. Higher Order Derivatives

$\frac{df}{dx}, \frac{d^2f}{dx^2}, \frac{d^3f}{dx^3}, \ldots$

$y = x^5$

$y' = 5x^4$

$y'' = 20x^3$

$y''' = 60x^2$

$y^{(4)} = 120x$

$y^{(5)} = 120$

$y^{(6)} = 0$

$y^{(7)} = 0$

$\vdots$